

Math 2080: Differential Equations

Worksheet 5.3: Discontinuous forcing terms

NAME:

The following properties of the Laplace transform will be useful in this worksheet:

$$(i) \mathcal{L}\{e^{at}\}(s) = \frac{1}{s-a}$$

$$(iii) \mathcal{L}\{\sin bt\}(s) = \frac{b}{s^2 + b^2}$$

$$(ii) \mathcal{L}\{t^n\}(s) = \frac{n!}{s^{n+1}},$$

$$(iv) \mathcal{L}\{y''(t)\}(s) = s^2 Y(s) - sy(0) - y'(0)$$

$$(v) \mathcal{L}\{f(t-c)H(t-c)\}(s) = e^{-cs} F(s)$$

1. Compute $\mathcal{L}\{(t-2)^2 H(t-2)\}(s)$.

2. Compute $\mathcal{L}\{t^2 H(t-2)\}(s)$.

3. Compute $\mathcal{L}\{e^{t-3} H(t-3)\}(s)$.

4. Compute $\mathcal{L}\{e^{t+3} H(t-3)\}(s)$.

5. Consider the initial value problem $y'' + y = f(t)$, $y(0) = 0$, $y'(0) = 1$, where $f(t) = \begin{cases} t, & 0 \leq t \leq 3 \\ 3, & t > 3 \end{cases}$

(a) Sketch $f(t)$, and write it using the Heavyside function.

(b) Take the Laplace transform of the differential equation, and solve for $Y(s)$.

- (c) Use partial fractions to decompose $Y(s)$ into four terms. [Note: $\frac{1}{s^2(s^2 + 1)} = \frac{1}{s^2} - \frac{1}{s^2 + 1}$.]
- (d) Apply the inverse Laplace transform to each term and write the solution to the IVP using the Heavyside function.
- (e) Write the solution as a piecewise function (i.e., *not* using the Heavyside function).