Lecture 3.1: Second order linear differential equations

Matthew Macauley

Department of Mathematical Sciences
Clemson University
http://www.math.clemson.edu/~macaule/

Math 2080, Differential Equations

Introduction

Definition

An equation of the form $y'' = f(t, y, y')$ is a second order differential equation. A solution is any function $y(t)$ such that

$$y''(t) = f(t, y(t), y'(t)) .$$

Motivating example

Newton's 2nd law of motion: $F = ma$. Force (could be gravitational, mechanical, etc.) can be a function of t (time), $x(t)$ (displacement), and $x'(t)$ (velocity). That is,

$$F = F(t, x, x') = mx''(t) .$$

Examples

Example 1

Gravitation force (constant).

Example 2

Spring force.

Example 3

Spring force plus gravity.

Example 4

Spring force plus gravity and damping.

Solving 2nd order ODEs

Two general techniques

(i) Solve them directly.

(ii) Convert into a system of two 1st order ODEs.

Solutions to 2nd order linear ODEs

Definition

A linear 2nd order ODE has the form $y'' + p(t)y' + q(t)y = f(t)$, and it is homogeneous if $f(t) = 0$.

Big idea

A linear 2nd order ODE has a 2-parameter family of solutions of the form

$$y(t) = C_1 y_1(t) + C_2 y_2(t) + y_p(t),$$

where $y_p(t)$ is *any* particular solution, and $y_1(t)$ and $y_2(t)$ solve the related "homogeneous equation."