Lecture 3.8: Power series solutions

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Introduction

Cauchy-Euler equations

Last time we looked at ODEs of the form $x^2y'' + axy' + by = 0$. It made sense that there would be a solution of the form $y(x) = x^r$.

Example 4

Consider the following homogeneous ODE: $y'' - 4xy' + 12y = 0$. Solve for $y(x)$.

Power series solutions

Example 4 (cont.)

Consider the following homogeneous ODE: $y'' - 4xy' + 12y = 0$. Solve for $y(x)$.

What do these solutions look like?

Example 4 (cont.)

The homogeneous ODE $y'' - 4xy' + 12y = 0$ has a power series solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$,

where the coefficients satisfy the following recurrence relation: $a_{n+2} = \frac{4(n-3)}{(n+2)(n+1)} a_n$.

Summary

The "power series method"

To solve $y'' - 4xy' + 12y = 0$ for $y(x)$, we took the following steps:

1. Assumed the solution has the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$.
2. Plugged the power series for $y(x)$ back into the ODE.
3. Combined into a single sum $y(x) = \sum_{n=0}^{\infty} [\quad \cdots \quad] x^n = 0$.
4. Set the x^n coefficient $[\cdots]$ equal to zero to get a recurrence $a_{n+2} = f(a_n)$.