# Lecture 4.7: Phase portraits with repeated eigenvalues

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Math 2080, Differential Equations

# Repeated eigenvalue, 2 eigenvectors

## Example 3a

Consider the following homogeneous system $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

# Repeated eigenvalue, 1 eigenvector

## Example 3b

Consider the following homogeneous system $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

# How to find a 2nd solution

## Example 3b

Consider the following homogeneous system $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

Since $\lambda_1 = -2$ and $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, we have a solution $\mathbf{x}_1(t) = e^{-2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

# Phase portrait

## Example 3b

Consider the following homogeneous system $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

The general solution is $\mathbf{x}(t) = C_1 e^{-2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{-2t} \left( t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$.