Lecture 5.2: Properties and applications of the Laplace transform

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Laplace transform fundamentals

Two key properties

- $\mathcal{L}$ is linear.
- $\mathcal{L}$ turns derivatives into multiplication.

Useful shortcuts

More properties

Suppose we know $F(s) = \mathcal{L}\{f(t)\}$. Then:

(i) $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$;

(ii) $\mathcal{L}\{tf(t)\} = -F'(s)$;

(iii) $\mathcal{L}\{t^n f(t)\} = (-1)^n \cdot \frac{d^n}{ds^n}F(s)$.

Examples

(i) Compute the Laplace transform of $f(t) = e^{2t}\cos 3t$.

(ii) Compute the Laplace transform of $f(t) = t^2 e^{3t}$.

Using the Laplace transform to solve ODEs

Example

Sove the initial value problem $y'' - y = e^{2t}$, $y(0) = 0$, $y'(0) = 1$.

Inverse Laplace transforms

Example

Compute $\mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+13}\right\}$.

Comparison of old vs. new methods

Structure of the solution to an ODE

A generic example

Consider an initial value problem $ay'' + by' + cy = f(t)$, $y(0) = x_0$, $y'(0) = v_0$.