Lecture 6.3: Fourier sine and cosine series

Matthew Macauley

Department of Mathematical Sciences
Clemson University
http://www.math.clemson.edu/~macaule/

Math 2080, Differential Equations

Exploiting symmetry

Definition

A function $f\colon \mathbb{R} \to \mathbb{R}$ is

- even if $f(x) = f(-x)$ for all $x \in \mathbb{R}$,
- odd if $f(x) = -f(-x)$ for all $x \in \mathbb{R}$.

Even and odd extensions

Definition

Let $f(x)$ be a function with domain $[0, \pi]$. There are several natural ways to make $f(x)$ periodic:

- the *periodic extension* of $f(x)$,
- the even extension of $f(x)$,
- the odd extension of $f(x)$.

Sine and cosine series

Definition

Let $f(x)$ be a function with domain $[0, \pi]$.

- The Fourier cosine series of f is the Fourier series of the even extension of f.
- The Fourier sine series of f is the Fourier series of the odd extension of f.

Computations

Example 1

Let $f(x) = x$ on $[0, \pi]$. Compute the Fourier sine and cosine series of $f(x)$.

Computations

Example 2

Compute the Fourier sine and cosine series of $f(x) = \begin{cases} x, & 0 \leq x < \pi/2 \\ \pi - x, & \pi/2 \leq x < \pi \end{cases}$

Save yourself some work

Example 3

Compute the Fourier sine series of the function $f(x) = x(\pi - x)$ defined on $[0, \pi]$.