Lecture 7.2: Different boundary conditions

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Math 2080, Differential Equations

Last time: Example 1a

The solution to the following IVP/BVP for the heat equation:

$$u_t = c^2 u_{xx}, \qquad u(0,t) = u(\pi,t) = 0, \qquad u(x,0) = x(\pi - x)\,.$$

is $u(x,t) = \displaystyle\sum_{n=1}^{\infty} \frac{4(1-(-1)^n)}{\pi n^3} \sin nx \, e^{-c^2 n^2 t}$.

This time: Example 1b

Solve the following IVP/BVP for the heat equation:

$$u_t = c^2 u_{xx}, \qquad u(0,t) = u(\pi,t) = 32, \qquad u(x,0) = x(\pi - x) + 32\,.$$

Last time: Example 1a

The solution to the following IVP/BVP for the heat equation:

$$u_t = c^2 u_{xx}, \qquad u(0,t) = u(\pi,t) = 0, \qquad u(x,0) = x(\pi - x)\,.$$

is $u(x,t) = \sum_{n=1}^{\infty} \frac{4(1-(-1)^n)}{\pi n^3} \sin nx \, e^{-c^2 n^2 t}$.

This time: Example 1c

Solve the following IVP/BVP for the heat equation:

$$u_t = c^2 u_{xx}, \qquad u(0,t) = 32, \quad u(\pi,t) = 42, \qquad u(x,0) = x(\pi - x) + 32 + \frac{10}{\pi} x\,.$$

A familiar theme

Summary

To solve the initial / boundary value problem

$$u_t = c^2 u_{xx}, \qquad u(0,t) = a, \quad u(\pi,t) = b, \qquad u(x,0) = h(x),$$

first solve the related homogeneous problem, then add this to the steady-state solution $u_{ss}(x,t) = a + \frac{b-a}{\pi}x$.

von Neumann boundary conditions (type 2)

Example 2

Solve the following IVP/BVP for the heat equation:

$$u_t = c^2 u_{xx}, \qquad u_x(0,t) = u_x(\pi,t) = 0, \qquad u(x,0) = x(\pi - x)\,.$$

von Neumann boundary conditions (type 2)

Example 2 (cont.)

The general solution to the following BVP for the heat equation:

$$u_t = c^2 u_{xx}, \qquad u_x(0,t) = u_x(\pi,t) = 0, \qquad u(x,0) = x(\pi - x)\,.$$

is $u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \, e^{-c^2 n^2 t}$. Now, we'll solve the remaining IVP.