

## Math 2080: Differential Equations

### Worksheet 3.5: Damped harmonic motion

**NAME:**

1. For the following exercises, rewrite the given function in the form

$$y = A \cos(\omega t - \phi) = A \cos\left(\omega\left(t - \frac{\phi}{\omega}\right)\right),$$

and then plot the graph of this function.

(a)  $y = 3 \cos 2t + 4 \sin 2t$

(b)  $y = 3 \cos 2t - 4 \sin 2t$

(c)  $y = -3 \cos 2t - 4 \sin 2t$

2. Consider the undamped oscillator

$$mx'' + kx = 0, \quad x(0) = x_0, \quad x'(0) = v_0.$$

Write the general solution of this initial value problem in the form  $x(t) = a \cos \omega t + b \sin \omega t$  (i.e., determine  $a$ ,  $b$ , and  $\omega$ ), and then write it in the form  $x(t) = A \cos(\omega t - \phi)$  (i.e., determine  $A$ ).

3. The function  $x(t) = \cos 6t - \cos 7t$  has mean frequency  $\bar{\omega} = 13/2$  and half difference  $\delta = 1/2$ . Thus,

$$\cos 6t - \cos 7t = \cos\left(\frac{13}{2} - \frac{1}{2}\right)t - \cos\left(\frac{13}{2} + \frac{1}{2}\right)t = 2 \sin \frac{1}{2}t \sin \frac{13}{2}t.$$

Use a computer or calculator to plot both  $f(t) = \cos 6t - \cos 7t$  and the “envelope”  $g(t) = 2 \sin \frac{1}{2}t$  on the same set of axes.

4. Let  $\omega_0 = 11$ . Use a computer to plot the graph of the function

$$x(t) = \frac{\cos \omega t - \cos \omega_0 t}{\omega_0^2 - \omega^2}$$

for  $\omega = 9, 10, 10.5, 10.9$ , and  $10.99$  on the time interval  $[0, 24]$ . Explain how these solutions approach the resonance solution as  $\omega \rightarrow \omega_0$ .