

## Math 2080: Differential Equations

### Worksheet 3.6: Variation of parameters

**NAME:**

Consider the inhomogeneous differential equation  $y'' + 9y = \sec 3t$ .

(a) Solve the related homogeneous equation,  $y_h'' + 9y_h = 0$ , by inspection.

(b) If  $y_h(t) = C_1y_1(t) + C_2y_2(t)$  is your answer to the previous part, then assume that there is a particular solution to the original ODE of the form  $y_p = v_1y_1 + v_2y_2$ . Compute  $y_p'$ .

(c) The  $y_p'$  from the previous part should have four terms; one each with  $v_1$ ,  $v_2$ ,  $v_1'$ , and  $v_2'$ . Eliminate the two terms involving  $v_1'$  and  $v_2'$  by setting their sum equal to zero. This leaves just two terms in  $y_p'$ . Now compute  $y_p''$ .

- (d) Plug  $y_p = v_1 y_1 + v_2 y_2$  and  $y_p''$  from Part (c) back into the original ODE. Things should cancel, leaving you with an equation involving  $v_1'$  and  $v_2'$  (no  $v_1$  or  $v_2$ ).
- (e) Write a system of two first order ODEs involving  $v_1'$  and  $v_2'$ . One equation comes from Part (c) and the other from Part (d). Write this in matrix form as well.
- (f) Solve for  $v_1'$  and  $v_2'$  by multiplying your two equations by suitable functions and and or subtracting them to eliminate the other.
- (g) Integrate to find  $v_1$  and  $v_2$  (feel free to use a computer). Write the general solution to the original ODE.