Math 2080: Differential Equations
Worksheet 4.3: Mixing with two tanks

NAME:

Tank A contains 10 gallons of a solution in which 5 oz of salt are dissolved. Tank B contains 20 gallons in which 6 oz of salt are dissolved. Salt water with a concentration of 2 oz/gal flows into each tank at a rate of 4 gal/min. The fully mixed solution drains from Tank A at a rate of 3 gal/min and from Tank B at a rate of 5 gal/min. Solution flows from Tank A to Tank B at a rate of 1 gal/min. Let $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, where $x_1(t)$ (respectively, $x_2(t)$) is the amount of salt in Tank A (resp., Tank B) after time t.

(a) Write down a system of ODEs (including the initial condition $\mathbf{x}(0)$) that models this situation, and write it in matrix form: $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}, \quad \mathbf{x}(0) = \mathbf{c}$.

(b) What is the steady-state solution, $\mathbf{x}_{ss}$?

(c) If $\mathbf{x}_{ss} = \begin{bmatrix} a \\ b \end{bmatrix}$, then change variables by setting $y_1 = x_1 - a$ and $y_2 = x_2 - b$. Plug y_1 and y_2 back into the system to get a related system in $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$. Don't forget the initial condition, $\boldsymbol{y}(0)$.