Math 2080: Differential Equations
Worksheet 4.6: Phase portraits with complex eigenvalues

NAME:

Consider the system of differential equations: $\begin{cases} x_1' = 3x_1 + 2x_2, & x_1(0) = 0 \\ x_2' = -5x_1 + x_2, & x_2(0) = 1 \end{cases}$

(a) Write this in matrix form, $\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x}$, $\boldsymbol{x}(0) = \boldsymbol{x}_0$.

(b) Given that the eigenvalues of $\boldsymbol{A}$ are $\lambda_1 = 2 + 3i$ and $\lambda_2 = 2 - 3i$, with associated eigenvectors $\boldsymbol{v}_1 = \begin{bmatrix} -1 - 3i \\ 5 \end{bmatrix}$ and $\boldsymbol{v}_2 = \begin{bmatrix} -1 + 3i \\ 5 \end{bmatrix}$, write the general solution to $\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x}$.

(c) Write the general solution as a linear combination of *real-valued* functions: $\boldsymbol{x}(t) = C_1\boldsymbol{u}(t) + C_2\boldsymbol{w}(t)$.

(d) Find the particular solution satisfying the initial condition.

(e) The phase portrait will consist of spiraling ellipses. To determine whether the spirals are clockwise ore counterclockwise, compute the $\boldsymbol{x}'(0) = \begin{bmatrix} x_1'(0) \\ x_2'(0) \end{bmatrix}$ and see which direction it points.

(f) Sketch the phase portrait of the system, and sketch the particular solution satisfying the initial condition. Feel free to use a compute to plot the approximate tilt and shape of the ellipse.