Math 2080: Differential Equations
Worksheet 4.8: Stability of phase portraits

NAME:

In this problem, consider the system of differenetiation equations $\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x}$, where $\boldsymbol{A} = \begin{bmatrix} \alpha & 1 \\ -1 & \alpha \end{bmatrix}$ and α is a parameter.

(a) Determine the eigenvalues of $\boldsymbol{A}$ in terms of α.

(b) Find the critical value or values of α where the qualitative nature of the phase portrait for the system changes.

(c) Draw a phase portrait for a value of α slight below, and for another value slightly above, each critical value.

(d) Draw a phase portrait when α is exactly the critical value.

(e) Summarize Parts (b)–(d) by dividing the number line below into regions corresponding to phase portraits of the same types. Clearly label your diagram.