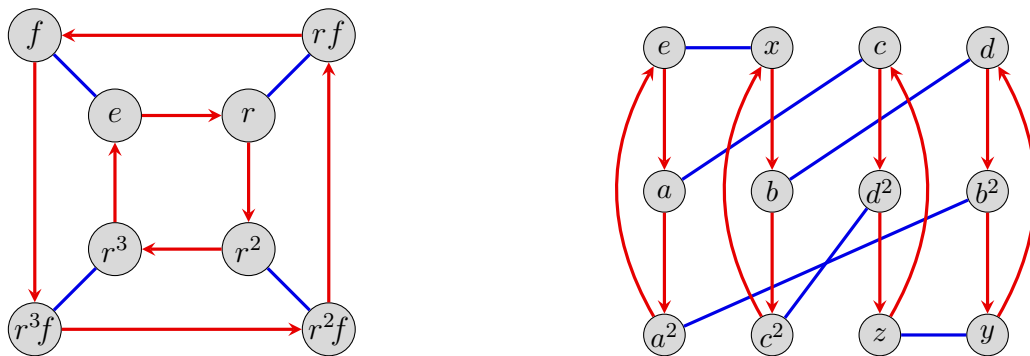


Read the following, which can all be found either in the textbook or on the course website.

- Chapters 3 & 4 of *Visual Group Theory* (VGT).
- VGT Exercises 3.5–3.10, 3.12. 4.3–4.5, 4.10–4.14.

Write up solutions to the following exercises.

1. Draw a Cayley diagram for each of the 7 frieze groups using a minimal generating set. Make it clear what your generators are and write out a group presentation for each.
2. Shown below are the Cayley graphs of two groups: D_4 is on the left, and the right is called an “alternating group,” denoted A_4 .



- (a) Create a multiplication table for each group. For consistency, order the elements in D_4 by $(e, r, r^2, r^3, f, rf, r^2f, r^3f)$ and by $(e, x, y, z, a, a^2, b, b^2, c, c^2, d, d^2)$ in A_4 .
 - (b) Find the inverse of each element of each group.
 - (c) Write out a group presentation for each group using the generators shown in the Cayley graph.
3. In each of the following multiplication tables, let e denote the identity element. Complete each table so its depicts a group. There may be more than one way to complete a table, in which case you need to give all possibilities.

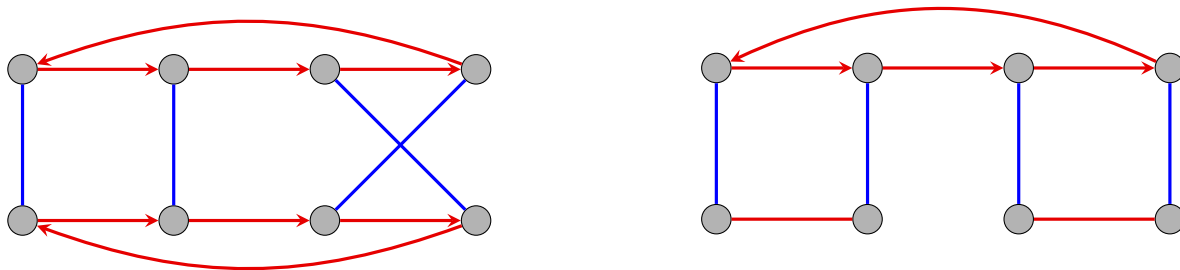
	e	a
e		
a		

	e	a	b
e			
a			
b			

	e	a	b	c
e				
a		e		
b				
c				

4. Prove that an element cannot appear twice in the same column of a multiplication table.
5. Using our “unofficial” definition of a group, prove that every group has a unique identity action e , satisfying $ge = g = eg$ for every action g in G . [Hint: You need to prove both existence and uniqueness. For the latter, assume that e and f are both identity actions. Can you prove that $e = f$?]

6. In this exercise, you will see one reason why our unofficial definition of a group doesn't quite match our formal definition. Consider the two diagrams below, neither of which is a valid Cayley diagram.



- Assuming that the arrows represent actions in a group, are the four rules in our unofficial definition of a group satisfied? For each rule, give a brief explanation why or why not.
 - Try to convert each of these diagrams into a multiplication table. What problem arises in each case?
 - Try to write a group presentation describing each “group.” What problem arises in each case?
 - Explain what is wrong with these diagrams that caused the problems you encountered in Parts (b) and (c)?
 - Create another diagram that is not a valid Cayley diagram for the same reason.
7. Let \mathbb{Z} , \mathbb{Q} , and \mathbb{R} denote the set of integers, rational numbers, and real numbers, respectively. Let \mathbb{Z}^+ , \mathbb{Q}^+ , and \mathbb{R}^+ denote the positive integers, rationals, and reals. Let \mathbb{Z}^* , \mathbb{Q}^* , and \mathbb{R}^* denote the nonzero integers, rationals, and reals.
- Which of the above sets are groups under addition? For each one that is a group (excluding \mathbb{R} , \mathbb{R}^+ , and \mathbb{R}^*), give a minimal generating set. For each one that is not, give an explicit reason for why it fails.
 - Which of the above sets are groups under multiplication? For each one that is a group (excluding \mathbb{R} , \mathbb{R}^+ , and \mathbb{R}^*), give a minimal generating set if there is one. For each one that is not, give an explicit reason for why it fails.
 - Let $n\mathbb{Z}$ denote the set of all integers that are multiples of n . For what n is the set $n\mathbb{Z}$ a group under addition? Give a minimal generating set for each one that is a group.