Lecture 1.1: An Introduction to Ordinary Differential Equations

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Math 2080, Differential Equations

Introduction to ODEs

What is a Differential Equation?

It is an equation involving a function and its derivatives.

Example (finance)

The rate of growth of an investment is *proportional* to the amount of the investment.

Equation: $P'(t) = rP(t)$. (Often, we just write $P' = rP$.)

For example, consider a mutual fund that grows at a 10% rate.

Note: We assume that interest is compounded *continuously*, i.e., at any point in time, the rate of change is $\frac{1}{10}P(t)$.

Modeling with ODEs

Big idea

If the rate of change of a function f is proportional to the function itself, then $f' = rf$.

Example (biology)

A colony of rabbits grows at a rate proportional to its size.

Modeling with to ODEs

Example (chemistry)

A radioactive substance decays at a rate proportional to its size.

Sample question: If there are 30 grams initially, and 20 grams after one year, what is the half-life?

Modeling with ODEs

Example (physics)

The temperature of a cup of coffee cools at a rate proportional to the difference: "(temp. of coffee) – (ambient temp.)".

Exponential decay

What else exhibits this "decay to a limiting value" behavior in nature (approximately)?

- Earth's population.
- Velocity of a falling object with air resistance.

Common theme: a family of solutions

Some questions from calculus:

- What is the antiderivative of $f(t) = 2t$?
- The velocity of a car is $x'(t) = 2t$. How far from home is it after t hours?
- An investment takes 5 years to double. How much is it worth after 8 years?