Lecture 2.8: The logistic equation

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Motivation

Recall

The ODE for exponential growth is $y'(t) = ry(t)$. *The rate r does not depend on $y(t)$.*

A better model of population growth

Suppose $y(t) =$ population of a colony.

- When $y(t)$ is small, it grows exponentially.
- When $y(t)$ is large, it grows slowly (decays to the carrying capacity, M).
- When $y(t)$ is too large, it decreases.

Key point: In general, the "rate" r *decreases* as $y(t)$ increases.

The logistic equation

Definition

The differential equation $y' = ry\left(1 - \frac{y}{M}\right)$ is called the logistic equation. The constant M represents the carrying capacity of the population.

An example

Example 1

The mass of a colony of bacteria grows according to the logistic equation. The colony lives in a petri dish that can hold 50 grams. Initially, there are 10 grams, and the mass is increasing at a rate of 1 gram per day. Find $m(t)$.

The threshold equation

Question

What if we replace r with $-r$ in the logistic equation?