Lecture 6.6: Boundary value problems

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Introduction

Initial vs. boundary value problems

If $y(t)$ is a function of time, then the following is an initial value problem (IVP):

$$y'' + 2y'' + 2y = 0, \qquad y(0) = 1, \quad y'(0) = 0$$

If $y(x)$ is a function of position, then the following is a boundary value problem (BVP):

$$y'' + 2y'' + 2y = 0, \qquad y(0) = 0, \quad y(\pi) = 0$$

The theory (existence and unique of solutions) of IVPs is well-understood. In contrast, BVPs are more complicated.

Solutions to boundary value problems

Examples

Solve the following boundary value problems:

1. $y'' = -y$, $y(0) = 0$, $y(\pi) = 0$.
2. $y'' = -y$, $y(0) = 0$, $y(\pi/2) = 0$.
3. $y'' = -y$, $y(0) = 0$, $y(\pi) = 1$.

Dirichlet boundary conditions (1st type)

Example 1

Find all solutions to the following boundary value problem:

$$y'' = \lambda y, \qquad y(0) = 0, \quad y(\pi) = 0 \,.$$

von Neumann boundary conditions (2nd type)

Example 2

Find all solutions to the following boundary value problem:

$$y'' = \lambda y, \qquad y'(0) = 0, \quad y'(\pi) = 0.$$

Mixed boundary conditions

Example 3

Find all solutions to the following boundary value problem:

$$y'' = \lambda y, \qquad y(0) = 0, \quad y'(\pi) = 0.$$