Lecture 7.3: The transport equation

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Math 2080, Differential Equations

Motivation

Some common one-dimensional PDEs

We've seen the heat equation: $u_t = c^2 u_{xx}$. In this lecture, we will introduce the transport equation, from which we will derive the wave equation: $u_{tt} = c^2 u_{xx}$.

Transport left

Example 1

Consider the following PDE involving a function $u(x, t)$:

$$\frac{\partial u}{\partial t} - c\frac{\partial u}{\partial x} = 0.$$

Transport right

Example 2

Consider the following PDE involving a function $u(x, t)$:

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0.$$

The wave equation

Example 3

Consider the following PDE involving a function $u(x, t)$:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} - c\frac{\partial}{\partial x}\right)u = \frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0$$

The two most common one-dimensional PDEs

Summary

Let $u(x,t)$ be a function of position x and time t. Then

- the heat equation is $u_t = c^2 u_{xx}$,
- the wave equation is $u_{tt} = c^2 u_{xx}$.