Lecture 7.4: The wave equation

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The goal

The wave equation

Recall that the wave equation is $u_{tt} = c^2 u_{xx}$. In this lecture, we will impose boundary and initial conditions and solve the resulting IVP/BVP.

Finding the general solution to the BVP

Example 3

Solve the following IVP/BVP for the wave equation:

$$u_{tt} = c^2 u_{xx}, \qquad u(0,t) = u(\pi,t) = 0, \qquad u(x,0) = x(\pi - x), \quad u_t(x,0) = 1\,.$$

Solving the initial value problem

Example 3 (cont.)

The general solution to the following BVP for the wave equation:

$$u_{tt} = c^2 u_{xx}, \qquad u(0,t) = u(\pi,t) = 0, \qquad u(x,0) = x(\pi - x), \quad u_t(x,0) = 1\,.$$

is $u(x,t) = \sum_{n=1}^{\infty} (a_n \cos cnt + b_n \sin cnt) \sin nx$. Now, we'll solve the remaining IVP.