Math 2080: Differential Equations
Worksheet 7.2: Different boundary conditions

NAME:

1. Let $u(x,t)$ be the temperature of a bar of length 10 that is fully insulated so that no heat can enter or leave, even at the endpoints. Suppose that initially, the temperature is increasing linearly from 70° at one endpoint to 80° at the other endpoint.

   (a) Sketch the initial heat distribution on the bar and express it as a function of x.

   (b) Write down an IVP/BVP to which $u(x,t)$ is a solution. (Let the constant from the heat equation be c^2.)

   (c) What will the steady-state solution be?

2. Let $u(x,t)$ be the temperature of a bar of length 10 that is fully insulated at its right endpoint but uninsulated at its left endpoint. Suppose the bar is sitting in a 70° room and that initially, the temperature of the bar increases linearly from 70° at the left endpoint to 80° at the other end. Finally, suppose the interior of the bar is poorly insulated, so heat can escape.

   (a) Suppose that heat escapes at a constant rate of 1° per hour. Write an IVP/BVP for $u(x,t)$ that could model this situation.

   (b) A more realistic situation would be for heat to escape not at a constant rate, but at a rate proportional to the *difference* between the temperature of the bar and the ambient temperature of the room. Write an IVP/BVP for $u(x,t)$ that could model this situation. What is the steady-state solution and why?

3. Consider the following PDE, where γ is a constant, and $h(x)$ and arbitrary function on $[0,\pi]$:

$$u_t = c^2 u_{xx}, \qquad u(0,t) = 0, \quad u_x(\pi,t) + \gamma\, u(\pi,t) = 0, \qquad u(x,0) = h(x)\ ,$$

   (a) Describe a physical situation that this models. Be sure to describe the impact of the initial condition, both boundary conditions, and the constant γ.

   (b) What is the steady-state solution and why? (Use your physical intuition).