Math 2080: Differential Equations
Worksheet 8.3: Predator-prey models

NAME:

(a) Consider a predator-prey interaction where the prey grows logistically and the predator would decay exponentially without the prey:

$$\begin{cases} X' = rX(1 - X/M) - sXY = rX(1 - \frac{1}{M}X - \frac{s}{r}Y) \\ Y' = Y(-u + vX). \end{cases}$$

Give a qualitative sketch of the X-nullclines and the Y-nullclines on the same set of axes. Mark the three steady-state solutions, which are intersections of nullclines.

(b) Now, consider a situation where the both species are valauable sources of food, and are harvested by an external species, such as humans. In this case, we can introduce terms $-E_1X$ in the prey equation and $-E_2Y$ in the predator equation. Consider the following example of this:

$$\begin{cases} X' = rX(1 - X/M) - sXY - E_1X = rX(1 - \frac{1}{M}X - \frac{s}{r}Y - E_1) \\ Y' = Y(-u + vX) - E_2Y = Y(-u + vX - E_2). \end{cases}$$

How do the nullclines, and consequentally, the steady-state solutions change if the prey is harvested but not the predator ($E_1 > 0$, $E_2 = 0$)? Include a qualitative sketch, and describe how it compares to Part (a).

(c) How do the nullclines, and consequentally, the steady-state solutions change if the predator is harvested but not the prey ($E_1 = 0$, $E_2 > 0$)? Include a qualitative sketch, and describe how it compares to Part (a).

(d) How do the nullclines, and consequentally, the steady-state solutions change if the prey and predator are harvested ($E_1 > 0$, $E_2 > 0$)? Include a qualitative sketch, and describe how it compares to Part (a).