Read the following, which can all be found either in the textbook or on the course website.

- Chapters 6 of *Visual Group Theory* (VGT).
- VGT Exercises 6.6–6.9, 6.12, 6.17–6.20, 6.28–6.30.

Write up solutions to the following exercises.

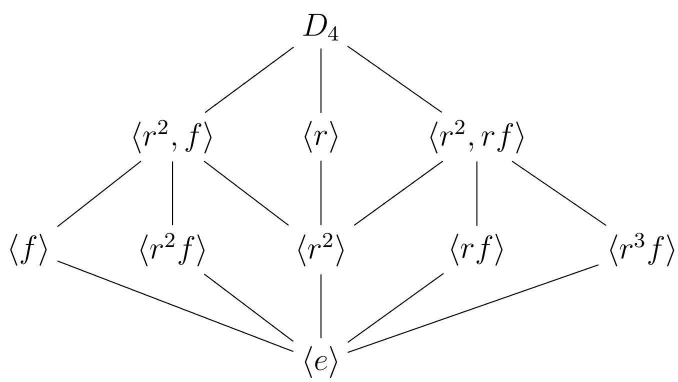
1. Consider the subgroups $H = \langle(1\ 2\ 3)\rangle$ and $K = \langle(1\ 2)(3\ 4)\rangle$ of the alternating group $G = A_4$. Carry out the following steps for both of these subgroups.

   (a) Write G as a disjoint union of the subgroup's left cosets.

   (b) Write G as a disjoint union of the subgroup's right cosets.

   (c) Determine whether the subgroup is normal in G.

2. The subgroup lattice of D_4 is shown below:



   For each of the 10 subgroups of D_4, determine whether it is normal. Justify each yes or no answer.

3. Consider a chain of subgroups $K \leq H \leq G$.

   (a) Prove or disprove: If $K \lhd H \lhd G$, then $K \lhd G$.

   (b) Prove or disprove: If $K \lhd G$, then $K \lhd H$.

4. The *center* of a group G is the set

$$\begin{aligned} Z(G) &= \{z \in G \mid gz = zg,\ \forall g \in G\} \\ &= \{z \in G \mid gzg^{-1} = z,\ \forall g \in G\}. \end{aligned}$$

   Prove that $Z(G)$ is a subgroup of G, and that it is normal in G.

5. Let H be a subgroup of G. Given two fixed elements $a, b \in G$, define the sets

$$aHbH = \{ah_1bh_2 \mid h_1, h_2 \in H\} \qquad \text{and} \qquad abH = \{abh \mid h \in H\}.$$

   Prove that if $H \lhd G$, then $aHbH = abH$.

6. Prove that $A \times \{e_B\}$ is a normal subgroup of $A \times B$, where e_B is the identity element of B. That is, show first that it is a subgroup, and then that it is normal.