Lecture 4.1: Boundary value problems

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Introduction

Initial vs. boundary value problems

If $y(t)$ is a function of time, then the following is an initial value problem (IVP):

$$y'' + 2y' + 2y = 0, \qquad y(0) = 1, \quad y'(0) = 0.$$

If $y(x)$ is a function of position, then the following is a boundary value problem (BVP):

$$y'' + 2y' + 2y = 0, \qquad y(0) = 0, \quad y(\pi) = 0$$

The theory (existence and unique of solutions) of IVPs is well-understood. In contrast, BVPs are more complicated.

Solutions to boundary value problems

Examples

Solve the following boundary value problems:

1. $y'' = -y, \; y(0) = 0, \; y(\pi) = 0.$
2. $y'' = -y, \; y(0) = 0, \; y(\pi/2) = 0.$
3. $y'' = -y, \; y(0) = 0, \; y(\pi) = 1.$

Dirichlet boundary conditions (1st type)

Example 1

Find all solutions to the following boundary value problem:

$$y'' = \lambda y, \qquad y(0) = 0, \quad y(L) = 0\,.$$

von Neumann boundary conditions (2nd type)

Example 2

Find all solutions to the following boundary value problem:

$$y'' = \lambda y, \qquad y'(0) = 0, \quad y'(L) = 0\,.$$

Mixed boundary conditions

Example 3

Find all solutions to the following boundary value problem:

$$y'' = \lambda y, \qquad y(0) = 0, \quad y'(L) = 0\,.$$

More complicated boundary conditions

Example 4

Find all solutions to the following boundary value problem:

$$y'' = \lambda y, \qquad y(0) = 0, \quad y(L) + y'(L) = 0\,.$$