Topics: PDEs on unbounded domains.

1. Solve the initial value problem for the heat equation, where $-\infty < x < \infty$ and $t > 0$:

$$u_t = c^2 u_{xx}, \qquad u(x,0) = \begin{cases} 1 & |x| \le 1 \\ 0 & |x| > 1. \end{cases}$$

   Write your solution in terms of the erf function. Sketch the heat distribution for $t = 0$ and several larger values of t on the same set of axes.

2. Solve the initial value problem for the wave equation, where $-\infty < x < \infty$ and $t > 0$:

$$u_{tt} = c^2 u_{xx}, \qquad u(x,0) = \frac{1}{1 + x^2/4}, \qquad u_t(x,0) = 0.$$

   Then repeat for the following IVP:

$$u_{tt} = c^2 u_{xx}, \qquad u(x,0) = 0, \qquad u_t(x,0) = \frac{1}{1 + x^2/4}.$$

   Describe what each IVP models. Compare and contrast the differences in the solutions. For each one, sketch the function $u(x,t)$ for $t = 0$, and several larger values of t on the same set of axes.

3. Solve the I/BVP for the heat equation on the semi-infinite domain $x > 0$, $t > 0$:

$$u_t = c^2 u_{xx}, \qquad u(0,t) = 0, \qquad u(x,0) = 1.$$

   Sketch a graph of $u(x,t)$ for $t = 0$ and several larger values of t.