Read the following, which can all be found either in the textbook or on the course website.

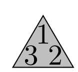
- Chapters 1–2 of *Visual Group Theory* (VGT), or Chapters 1–3 of *IBL Abstract Algebra*.
- VGT Exercises 1.1–1.4, 1.8–1.12, 2.13–2.17.
- The article *Group Think* by Steven Strogatz, which appeared in the NY Times in 2010.
- Francis Su's essay on good mathematical writing.

Write up solutions to the following exercises.

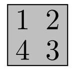
1. Answer the following questions after reading Francis Su's essay on good mathematical writing.
   (a) What is a good rule of thumb for what you should assume of your audience as you write your homework sets?
   (b) Is chalkboard writing formal or informal writing?
   (c) Why is the proof by contradiction on page 3 not really a proof by contradiction?
   (d) Name three things a lazy writer would do that a good writer would not.
   (e) What's the difference in meaning between these three phrases?

   "Let $A = 12$." "So $A = 12$." "$A = 12$."

2. Given a regular n-gon, let r be a rotation of it by $2\pi/n$ radians. This time, assume that we are not allowed to flip over the n-gon. These n actions form a group denoted $C_n = \langle r \rangle = \{e, r, r^2, \dots, r^{n-1}\}$.
   (a) Draw a Cayley diagram for C_n for $n = 4$, $n = 5$, and $n = 6$.
   (b) For $n = 4, 5, 6$, find all minimal generating sets of C_n. [*Note:* There are minimal generating sets of C_6 of size 2.]
   (c) Make a conjecture of what integers k does $C_n = \langle r^k \rangle$ for a general fixed integer n.

3. As we saw in lecture, the six symmetries of an equilateral triangle 

form a group denoted $D_3 = \{e, r, r^2, f, rf, r^2f\}$, where r is a 120° clockwise rotation and f is a flip about a vertical axis (which fixes the top corner). Since r and f suffice to generate all six of these symmetries, we write $D_3 = \langle r, f \rangle$.
   (a) Let g be the reflection of the triangle that fixes the lower-left corner. Which of the six actions in D_3 is g equal to? Which action is fg?
   (b) Write all 6 actions of D_3 using only f and g. Draw a Cayley diagram using f and g as generators.
   (c) To generate D_3, we need at least 2 actions. It is not difficult to show that if we have 3 generators, then one of them is unnecessary. Find all *minimal* generating sets of $D_3 = \{e, r, r^2, f, rf, r^2f\}$; note that all of them should have exactly two actions. Do not use g in this list.

4. The eight symmetries of a square 

form a group denoted D_4. Let r be a 90° clockwise rotation and f a horizontal flip (that is, about a vertical axis). It is not difficult to show that $D_4 = \langle r, f \rangle$.

   (a) Write all 8 actions of D_4 using r and f and draw a Cayley diagram using these two actions as generators.

   (b) Let g be the reflection of the square that fixes the lower-left and upper-right corner. Which of the eight actions in D_4 is g equal to? Which action is fg?

   (c) Draw a Cayley diagram of D_4 using f and g as generators.

   (d) Find all *minimal* generating sets of D_4. [*Hint*: There are 12.]

   (e) Find a minimal generating set of D_6 (the 12 symmetries of a regular hexagon) that has three actions.

5. Pick any integer and consider this set of actions: adding any integer to the one you choose. This is an infinite set of actions; we might name them like "add 1" and "add −4120," etc. This is a group.

   (a) Find all generating sets of size 1 and sketch a Cayley diagram for each.

   (b) Find a minimal generating set of size 2 and sketch the Cayley diagram.

   (c) Find a minimal generating set of size 3.

   (d) Explain how to construct a minimal generating set of size n, for any $n \in \mathbb{N}$.