Lecture 5.1: Symmetric cryptographic ciphers

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Math 4190, Discrete Mathematical Structures

Encoding messages as numbers

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Now, suppose that we wanted to send this as a secret message...

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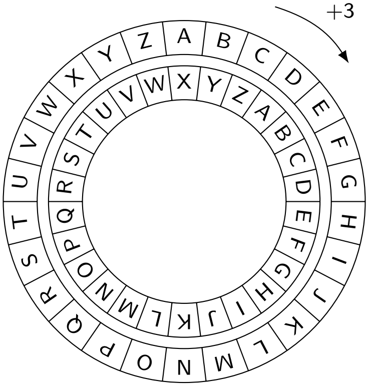

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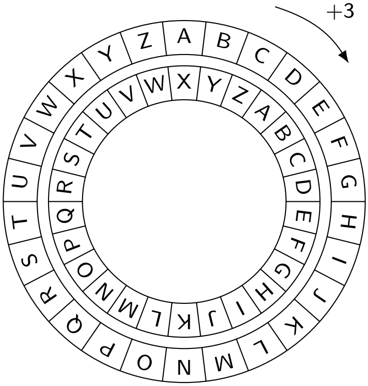

Decrypted message: **OW THATS SHARP**

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We encrypt the letter **R** by

$$\begin{aligned} e(17) &\equiv 17 + 18 \pmod{26} \\ &\equiv 35 \pmod{26} \\ &\equiv 9 \pmod{26} \end{aligned}$$

which is **J**.

Let's decrypt **L**:

Caesar cipher

The Caesar cipher is defined by the following:

- key, $k \in \mathbb{N}$,
- encryption function, $e(x)$,
- decryption function, $d(y)$,

$$e(x) = x + k \pmod{26}, \qquad d(y) = y - k \pmod{26}.$$

We first associate each letter to a number in $\mathbb{Z}_{26} = \{0, 1, \dots, 24, 25\}$, as follows:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
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Consider the following encryption function:

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Number theory fact

A number $k \in \mathbb{Z}_n$ has a multiplicative inverse iff $\gcd(n, k) = 1$.

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The decryption function is thus

$$d(x) = 21(x - 3) \pmod{26}.$$

A weakness of character ciphers

The ciphers that we've seen are called character, or monographic ciphers: all copies of the same letter get encrypted the same way:

$$e(x_i) = e(x_j) \ \Rightarrow x_i = x_j.$$

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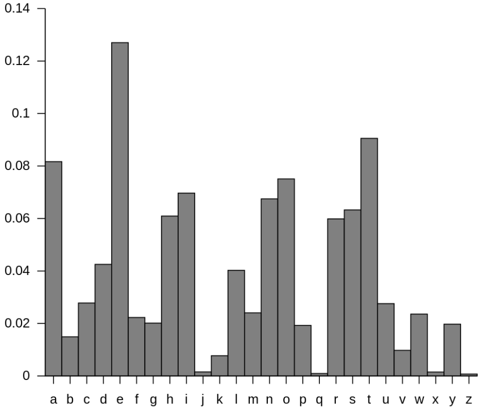
If the message is long, then the private key can be deduced by analyzing letter frequencies.



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Let's encrypt the message **ENGINEERING**:

$$p_1 p_2 p_3 p_4 p_5 p_6 p_7 p_8 p_9 p_{10} p_{11} = 4 \;\; 13 \;\; 6 \;\; 8 \;\; 13 \;\; 4 \;\; 4 \;\; 17 \;\; 8 \;\; 13 \;\; 6$$

using the key **ROCKS**:

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	E	**N**	**G**	**I**	**N**	**E**	**E**	**R**	**I**	**N**	**G**
p_i	4	13	6	8	13	4	4	17	8	13	6

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p_i	4	13	6	8	13	4	4	17	8	13	6
k_i	17	14	2	10	18	17	14	2	10	18	17
$c_i = p_i + k_i$	21	1	8	18	5	21	18	19	18	5	23

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p_i	4	13	6	8	13	4	4	17	8	13	6
k_i	17	14	2	10	18	17	14	2	10	18	17
$c_i = p_i + k_i$	21	1	8	18	5	21	18	19	18	5	23
	V	**B**	**I**	**S**	**F**	**V**	**S**	**T**	**S**	**F**	**X**

Decryption with the Vigenère cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Let's decrypt the message **TZGWK FBVSY WFU**:

$$c_1 c_2 c_3 c_4 c_5 c_6 c_7 c_8 c_9 c_{10} c_{11} c_{12} c_{13} = 19\ \ 25\ \ 6\ \ 22\ \ 10\ \ 5\ \ 1\ \ 21\ \ 18\ \ 24\ \ 22\ \ 5\ \ 20$$

using the same key **ROCKS**:

$$k_1 k_2 k_3 k_4 k_5 = 17\ \ 14\ \ 2\ \ 10\ \ 18.$$

Decryption with the Vigenère cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
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T	Z	G	W	K	F	B	V	S	Y	W	F	U

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	T	**Z**	**G**	**W**	**K**	**F**	**B**	**V**	**S**	**Y**	**W**	**F**	**U**
c_i	19	25	6	22	10	5	1	21	18	24	22	5	20

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c_i	19	25	6	22	10	5	1	21	18	24	22	5	20
k_i	17	14	2	10	18	17	14	2	10	18	17	14	2

Decryption with the Vigenère cipher

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	T	**Z**	**G**	**W**	**K**	**F**	**B**	**V**	**S**	**Y**	**W**	**F**	**U**
c_i	19	25	6	22	10	5	1	21	18	24	22	5	20
k_i	17	14	2	10	18	17	14	2	10	18	17	14	2
$p_i = c_i - k_i$	2	11	4	12	18	14	13	19	8	6	4	17	18

Decryption with the Vigenère cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Let's decrypt the message **TZGWK FBVSY WFU**:

$$c_1c_2c_3c_4c_5c_6c_7c_8c_9c_{10}c_{11}c_{12}c_{13} = 19\ \ 25\ \ 6\ \ 22\ \ 10\ \ 5\ \ 1\ \ 21\ \ 18\ \ 24\ \ 22\ \ 5\ \ 20$$

using the same key **ROCKS**:

$$k_1k_2k_3k_4k_5 = 17\ \ 14\ \ 2\ \ 10\ \ 18.$$

	T	**Z**	**G**	**W**	**K**	**F**	**B**	**V**	**S**	**Y**	**W**	**F**	**U**
c_i	19	25	6	22	10	5	1	21	18	24	22	5	20
k_i	17	14	2	10	18	17	14	2	10	18	17	14	2
$p_i = c_i - k_i$	2	11	4	12	18	14	13	19	8	6	4	17	18
	C	**L**	**E**	**M**	**S**	**O**	**N**	**T**	**I**	**G**	**E**	**R**	**S**

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The Rivest–Shamir–Adleman (RSA) cryptosystem is a popular asymmetric cryptosystem, which we'll learn about this in an upcoming lecture.