# Lecture 2.4: Undetermined coefficients

Matthew Macauley

Department of Mathematical Sciences
Clemson University
http://www.math.clemson.edu/~macaule/

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# The main idea

## Theorem

The general solution to $y' + a(t)y = g(t)$ has the form

$$y(t) = y_h(t) + y_p(t) = \Big\{ C_1 y_1(t) + y_p(t) \mid C_1 \in \mathbb{C} \Big\},$$

where $y_p(t)$ is *any particular solution* to the original ODE.

## Example 1

Solve the differential equation $y' + 2y = 6$.

# Polynomial forcing term

## Example 2

Solve $y' + 2y = 6t^2$.

# Exponential forcing term

## Example 3

Solve $y' + 2y = 6e^{3t}$.

# Sinusoidal forcing term

## Example 4

Solve $y' + 2y = 6\cos 3t$.

# A "problem case"

## Example 5

Solve $y' + 2y = 6e^{-2t}$.

# Solving linear inhomogeneous ODEs

## Summary

The technique we've been using is called the method of undetermined coefficients. It works as long as we can:

(i) Solve the homogeneous equation.

(ii) Find a solution by inspection or educated guess.

If we can't do (i), we're out of luck.

If we can't do (ii), then we have two alternative methods:

1. Integrating factor.
2. Variation of parameters.

These methods work as long as we can integrate $e^{\int a(t)}g(t)$.

# When you can't guess a particular solution

**Example 6**

Solve $y' + \frac{1}{t}y = 1$.

# 2nd order inhomogeneous ODEs

## Theorem

The general solution to $y'' + a(t)y' + b(t)y = g(t)$ has the form

$$y(t) = y_h(t) + y_p(t) = \Big\{ C_1 y_1(t) + C_2 y_2(t) + y_p(t) \mid C_1 \in \mathbb{C} \Big\},$$

where $y_p(t)$ is *any particular solution* to the original ODE.

To prove this, just show that $y - y_p$ solves the homogeneous equation. (HW)

## Method

To solve a 2nd order linear inhomogeneous ODE:

1. Solve the homogeneous equation: $y_h'' + a(t)y_h' + b(t)y_h = 0$.
2. Find *any* particular solution $y_p(t)$ to the original equation.
3. Add these together: $y(t) = y_h(t) + y_p(t) = C_1 y_1(t) + C_2 y_2(t) + y_p(t)$.

# 2nd order inhomogeneous ODEs

## More examples

(i) Solve $y'' + 3y' + 2y = 1$.

(ii) Solve $y'' + 3y' + 2y = -4t^2$.

(iii) Solve $y'' + 3y' + 2y = 4e^{3t}$.