Read the following, which can all be found either in the textbook or on the course website.

- Chapters 1–2 of Visual Group Theory (VGT), or Chapters 1–3 of IBL Abstract Algebra.
- VGT Exercises 1.1–1.4, 1.8–1.12, 2.13–2.17.
- The article *Group Think* by Steven Strogatz, which appeared in the NY Times in 2010.
- Francis Su's essay on good mathematical writing.

Write up solutions to the following exercises.

- 1. Answer the following questions after reading Francis Su's essay on good mathematical writing.
  - (a) What is a good rule of thumb for what you should assume of your audience as you write your homework sets?
  - (b) Is chalkboard writing formal or informal writing?
  - (c) Why is the proof by contradiction on page 3 not really a proof by contradiction?
  - (d) Name three things a lazy writer would do that a good writer would not.
  - (e) What's the difference in meaning between these three phrases?

"Let A = 12." "So A = 12." "A = 12."

- 2. Given a regular *n*-gon, let *r* be a rotation of it by  $2\pi/n$  radians. This time, assume that we are not allowed to flip over the *n*-gon. These *n* actions form a group denoted  $C_n = \langle r \rangle = \{e, r, r^2, \ldots, r^{n-1}\}.$ 
  - (a) Draw a Cayley diagram for  $C_n$  for n = 4, n = 5, and n = 6.
  - (b) For n = 4, 5, 6, find all minimal generating sets of  $C_n$ . [Note: There are minimal generating sets of  $C_6$  of size 2.]
  - (c) Make a conjecture of what integers k does  $C_n = \langle r^k \rangle$  for a general fixed integer n.
- 3. As we saw in lecture, the six symmetries of an equilaterial triangle form a group denoted  $D_3 = \{e, r, r^2, f, rf, r^2f\}$ , where r is a 120° clockwise rotation and f is a flip about a vertical axis (which fixes the top corner). Since r and f suffice to generate all six of these symmetries, we write  $D_3 = \langle r, f \rangle$ .
  - (a) Let g be the reflection of the triangle that fixes the lower-left corner. Which of the six actions in  $D_3$  is g equal to? Which action is fg?
  - (b) Write all 6 actions of  $D_3$  using only f and g. Draw a Cayley diagram using f and g as generators.
  - (c) To generate  $D_3$ , we need at least 2 actions. It is not difficult to show that if we have 3 generators, then one of them is unnecessary. Find all *minimal* generating sets of  $D_3 = \{e, r, r^2, f, rf, r^2 f\}$ ; note that all of them should have exactly two actions. Do not use g in this list.

- 4. The eight symmetries of a square  $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$  form a group denoted  $D_4$ . Let r be a 90° clockwise rotation and f a horizontal flip (that is, about a vertical axis). It is not difficult to show that  $D_4 = \langle r, f \rangle$ .
  - (a) Write all 8 actions of  $D_4$  using r and f and draw a Cayley diagram using these two actions as generators.
  - (b) Let g be the reflection of the square that fixes the lower-left and upper-right corner. Which of the eight actions in  $D_4$  is g equal to? Which action is fg?
  - (c) Draw a Cayley diagram of  $D_4$  using f and g as generators.
  - (d) Find all minimal generating sets of  $D_4$ . [Hint: There are 12.]
  - (e) Find a minimal generating set of  $D_6$  (the 12 symmetries of a regular hexagon) that has three actions.
- 5. Pick any integer and consider this set of actions: adding any integer to the one you choose. This is an infinite set of actions; we might name them like "add 1" and "add -4120," etc. This is a group.
  - (a) Find all generating sets of size 1 and sketch a Cayley diagram for each.
  - (b) Find a minimal generating set of size 2 and sketch the Cayley diagram.
  - (c) Find a minimal generating set of size 3.
  - (d) Explain how to construct a minimal generating set of size n, for any  $n \in \mathbb{N}$ .