

Topics: Linear maps.

Do: Answer the following questions. Throughout, assume that X is a finite-dimensional vector space over a field K .

1. Let $T: X \rightarrow U$ be a linear map.

(a) Show that the *image* $T(X)$ (i.e., the *range*, R_T) is a subspace of U .

(b) Show that the *preimage* of a subspace $V \leq U$, denoted

$$T^{-1}(V) := \{x \in X \mid Tx \in V\},$$

is a subspace of X .

2. Let $A: X \rightarrow X$ be a linear map.

(a) Prove that if $\dim(X) < \infty$, then the following are equivalent:

(i) A is bijective.

(ii) A is injective.

(iii) A is surjective.

(b) How can the results of Part (a) fail if $\dim(X) = \infty$? Give an explicit example to demonstrate this.