MTHSC 208 (Differential Equations)
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HW 16
Due Monday March 23rd, 2009

(1) Use the ratio test to find the radius of convergence of the following power series:
   a. $\sum_{n=0}^{\infty} (-1)^n x^n$,   b. $\sum_{n=0}^{\infty} \frac{1}{n+1} (x - \pi)^n$,   c. $\sum_{n=0}^{\infty} \frac{3}{n+1} (x - 2)^n$,
   d. $\sum_{n=0}^{\infty} \frac{1}{2^n} (x - \pi)^n$,   e. $\sum_{n=0}^{\infty} (5x - 10)^n$,   f. $\sum_{n=0}^{\infty} \frac{1}{n!} (3x - 6)^n$.

(2) Use the comparison test to find an estimate for the radius of convergence of each of the following power series:
   a. $\sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n}$,   b. $\sum_{n=0}^{\infty} (-1)^n x^{2n}$,
   c. $\sum_{n=1}^{\infty} \frac{1}{2^n} (x - 4)^{2n}$,   d. $\sum_{n=0}^{\infty} \frac{1}{2^n} (x - \pi)^{2n}$.

(3) Find the radius of convergence of the power series
   $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(n!)^2} \left(\frac{x}{2}\right)^{2n}$.

(4) The differential equation
   $$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0,$$
   where $p$ is a constant, is known as Chebyshev’s equation. It can be rewritten in the form
   $$\frac{d^2 y}{dx^2} - P(x) \frac{dy}{dx} + Q(x) y = 0,$$
   where $P(X) = -\frac{x}{1 - x^2}$, $Q(x) = \frac{p^2}{1 - x^2}$.

   (a) If $P(x)$ and $Q(x)$ are represented as a power series about $x_0 = 0$, what is the radius of convergence of these power series?
   (b) Assuming a power series centered at 0, find the recursion formula for $a_{n+2}$ in terms of $a_n$.
   (c) Use the recursion formula to determine $a_n$ in terms of $a_0$ and $a_1$, for $2 \leq n \leq 9$.
   (d) In the special case where $p = 3$, find a nonzero polynomial solution to this differential equation.