MTHSC 208 (Differential Equations)
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HW 16
Due Monday March 23rd, 2009

(1) Use the ratio test to find the radius of convergence of the following power series:

$$a.\ \sum_{n=0}^{\infty}(-1)^n x^n, \qquad b.\ \sum_{n=0}^{\infty}\frac{1}{n+1}(x-\pi)^n, \qquad c.\ \sum_{n=0}^{\infty}\frac{3}{n+1}(x-2)^n,$$

$$d.\ \sum_{n=0}^{\infty}\frac{1}{2^n}(x-\pi)^n, \qquad e.\ \sum_{n=0}^{\infty}(5x-10)^n, \qquad f.\ \sum_{n=0}^{\infty}\frac{1}{n!}(3x-6)^n.$$

(2) Use the comparison test to find an estimate for the radius of convergence of each of the following power series:

$$a.\ \sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n}, \qquad b.\ \sum_{n=0}^{\infty}(-1)^n x^{2n},$$

$$c.\ \sum_{n=1}^{\infty}\frac{1}{2n}(x-4)^{2n}, \qquad d.\ \sum_{n=0}^{\infty}\frac{1}{2^{2n}}(x-\pi)^{2n}.$$

(3) Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty}(-1)^n\frac{1}{(n!)^2}\left(\frac{x}{2}\right)^{2n}.$$

(4) The differential equation

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0,$$

where p is a constant, is known as Chebyshev's equation. It can be rewritten in the form

$$\frac{d^2y}{dx^2}-P(x)\frac{dy}{dx}+Q(x)y=0, \qquad \text{where} \qquad P(X)=-\frac{x}{1-x^2}, \quad Q(x)=\frac{p^2}{1-x^2}.$$

(a) If $P(x)$ and $Q(x)$ are represented as a power series about $x_0=0$, what is the radius of convergence of these power series?
(b) Assuming a power series centered at 0, find the recursion formula for a_{n+2} in terms of a_n.
(c) Use the recursion formula to determine a_n in terms of a_0 and a_1, for $2 \le n \le 9$.
(d) In the special case where $p=3$, find a nonzero polynomial solution to this differential equation.