MTHSC 208 (Differential Equations)
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HW 17
Due Friday March 27th, 2009

(1) For each of the following ODEs, determine whether $x = 0$ is an ordinary or singular point. If it is singular, determine whether it is regular or not. (Remember, first write each ODE in the form $y'' + P(x)y' + Q(x)y = 0$.)
   (a) $y'' + xy' + (1 - x^2)y = 0$
   (b) $y'' + (1/x)y' + (1 - (1/x^2))y = 0$.
   (c) $x^2y'' + 2xy' + (\cos x)y = 0$.
   (d) $x^3y'' + 2xy' + (\cos x)y = 0$.

(2) Consider the differential equation $3xy'' + y' + y = 0$. Since $x_0 = 0$ is a regular singular point, there is a solution of the form

$$y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}.$$

   (a) Determine the indicial equation (solve for r) and the recursion formula.
   (b) Find two linearly independent generalized power series solutions (i.e., a *basis* for the solution space).
   (c) Determine the radius of convergence of each of these solutions. (Hint: First compute the radius of convergence of $xP(x)$ and $x^2Q(x)$ and apply Frobenius).

(3) Consider the differential equation $2xy'' + y' + xy = 0$. Since $x_0 = 0$ is a regular singular point, there is a solution of the form

$$y(x) = \sum_{n=0}^{\infty} a_n x^{n+r} .$$

   (a) Determine the indicial equation (solve for r) and the recursion formula.
   (b) Find two linearly independent generalized power series solutions.
   (c) What is the radius of convergence of these solutions?

(4) Consider the differential equation $xy'' + 2y' - xy = 0$.
   (a) Show that $x = 0$ is a regular singular point.
   (b) Show that if $a_0 = 0$, then $r = -1$ is one solution for the indicial equation.
   (c) For $r = -1$ and $a_0 = 0$, find the recurrence relation for a_{n+2} in terms of a_n.
   (d) Still assuming that $a_0 = 0$, write the solution from (b) as a generalized power series.
   (e) Which elementary function (i.e., standard everyday function) is this solution?