MTHSC 208 (Differential Equations)
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HW 18
Due Monday March 30th, 2009

(1) For each of the following sets, determine if it is a vector space. If it is, give a basis. If it isn’t, explain why not.
   (a) The set of points in $\mathbb{R}^3$ with $x = 0$.
   (b) The set of points in $\mathbb{R}^2$ with $x = y$.
   (c) The set of points in $\mathbb{R}^3$ with $x = y$.
   (d) The set of points in $\mathbb{R}^3$ with $z \geq 0$.
   (e) The set of unit vectors in $\mathbb{R}^2$.
   (f) The set of polynomials of degree $n$.
   (g) The set of polynomials of degree at most $n$.
   (h) The set of polynomials of degree at most $n$, with only even-powers of $x$.
   (i) The set of generalized power series of the form $\sum_{n=0}^{\infty} a_n x^{n-\frac{3}{2}}$.
   (j) The set of $2\pi$-periodic functions.

(2) Complete the following sentences:
   (a) Two non-zero vectors $v_1$ and $v_2$ are a basis for $\mathbb{R}^2$ if and only if... 
   (b) Three non-zero vectors $v_1$, $v_2$, and $v_3$ are a basis for $\mathbb{R}^3$ if and only if...

(3) Let $X$ be a vector space over $\mathbb{C}$ (i.e., the constants are complex numbers, instead of just real numbers). If $\{v_1, v_2\}$ is a basis of $X$, then by definition, every vector $v$ can be written uniquely as $v = C_1 v_1 + C_2 v_2$.
   (a) Is the set $\{v_1 + v_2, 3v_1 - 2v_2\}$ also a basis of $X$?
   (b) Is the set $\{\frac{1}{2}v_1 + \frac{1}{2}v_2, \frac{1}{2}v_1 - \frac{1}{2}v_2\}$ a basis of $X$?
   (c) Consider the ODE $y'' + 4y = 0$. If we assume that $y(t) = e^{rt}$, then we get that $r = \pm 2i$. Therefore, the general solution is $y(t) = C_1 e^{2it} + C_2 e^{-2it}$, i.e., $\{e^{2it}, e^{-2it}\}$ is a basis for the solution space. Use (b), and Euler’s equation ($e^{i\theta} = \cos \theta + i \sin \theta$) to find a basis for the solution space involving sines and cosines, and write the general solution using sines and cosines.