MTHSC 208 (Differential Equations)
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HW 18
Due Monday March 30th, 2009

(1) For each of the following sets, determine if it is a vector space. If it is, give a basis. If it isn't, explain why not.
  (a) The set of points in $\mathbb{R}^3$ with $x = 0$.
  (b) The set of points in $\mathbb{R}^2$ with $x = y$.
  (c) The set of points in $\mathbb{R}^3$ with $x = y$.
  (d) The set of points in $\mathbb{R}^3$ with $z \geq 0$.
  (e) The set of unit vectors in $\mathbb{R}^2$.
  (f) The set of polynomials of degree n.
  (g) The set of polynomials of degree at most n.
  (h) The set of polynomials of degree at most n, with only even-powers of x.
  (i) The set of generalized power series of the form $\sum_{n=0}^{\infty} a_n x^{n-\frac{2}{3}}$.
  (j) The set of 2π-periodic functions.

(2) Complete the following sentences:
  (a) Two non-zero vectors v_1 and v_2 are a basis for $\mathbb{R}^2$ if and only if...
  (b) Three non-zero vectors v_1, v_2, and v_3 are a basis for $\mathbb{R}^3$ if and only if...

(3) Let X be a vector space over $\mathbb{C}$ (i.e., the contants are complex numbers, instead of just real numbers). If $\{v_1, v_2\}$ is a basis of X, then by definition, every vector v can be written uniquely as $v = C_1 v_1 + C_2 v_2$.
  (a) Is the set $\{v_1 + v_2, 3v_1 - 2v_2\}$ also a basis of X?
  (b) Is the set $\{\frac{1}{2}v_1 + \frac{1}{2}v_2, \frac{1}{2i}v_1 - \frac{1}{2i}v_2\}$ a basis of X?
  (c) Consider the ODE $y'' + 4y = 0$. If we assume that $y(t) = e^{rt}$, then we get that $r = \pm 2i$. Therefore, the general solution is $y(t) = C_1 e^{2it} + C_2 e^{-2it}$, i.e., $\{e^{2it}, e^{-2it}\}$ is a basis for the solution space. Use (b), and Euler's equation ($e^{i\theta} = \cos\theta + i\sin\theta$) to find a basis for the solution space involving sines and cosines, and write the general solution using sines and cosines.