MTHSC 208 (Differential Equations)
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HW 20
Due Monday April 6th, 2009

(1) Determine which of the following functions are even, which are odd, and which are neither even nor odd:
  (a) $f(t) = t^3 + 3t$.
  (b) $f(t) = t^2 + |t|$.
  (c) $f(t) = e^t$.
  (d) $f(t) = \frac{1}{2}(e^t + e^{-t})$.
  (e) $f(t) = \frac{1}{2}(e^t - e^{-t})$.

(2) Suppose that f is a function defined on $\mathbb{R}$ (not necessarily periodic). Show that there is an odd function f_{odd} and an even function f_{even} such that $f(x) = f_{\text{odd}} + f_{\text{even}}$. *Hint*: Look at Problem (1c), (d), and (e) together.

(3) (a) The Fourier series of an odd function consists only of sine-terms. What additional symmetry conditions on f will imply that the sine coefficients with even indices will be zero (i.e., each $b_{2n} = 0$)? Give an example of a non-zero function satisfying this additional condition.
  (b) What symmetry conditions on f will imply that the sine coefficients with odd indices will be zero (i.e., each $b_{2n+1} = 0$)? Give an example of a non-zero function satisfying this additional condition.
  (c) Sketch the graph of a non-zero even function, such that $a_{2n} = 0$ for all n.
  (d) Sketch the graph of a non-zero even function, such that $a_{2n+1} = 0$ for all n.

(4) Express the y-intercept of $f(t) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$ in terms of the a_n's and b_n's. (*Hint*: It's *not* a_0 or $a_0/2$!)

(5) Consider the 2π-periodic function $f(t) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$. Write the Fourier series for the following functions:
  (a) The reflection of $f(t)$ across the y-axis;
  (b) The reflection of $f(t)$ across the x-axis;
  (c) The reflection of $f(t)$ across the origin.

(6) Consider the function defined on the interval $[0, \pi]$:

$$f(t) = \begin{cases} t & \text{for } 0 \le t < \pi/2, \\ \pi - t, & \text{for } \pi/2 \le t \le \pi. \end{cases}$$

  (a) Sketch the even extension of this function and find its Fourier cosine series.
  (b) Sketch the odd extension of this function and find its Fourier sine series.

(7) Consider the function defined on the interval $[0, \pi]$:

$$f(t) = t(\pi - t).$$

  a. Sketch the even extension of this function and find its Fourier cosine series.
  b. Sketch the odd extension of this function and find its Fourier sine series.

(8) (a) Find the complex Fourier coefficients of the function

$$f(t) = t^2 \qquad \text{for } -\pi < t \le \pi,$$

extended to be periodic of period 2π.

(b) Find the real form of the Fourier series. *Hint: Use* $a_n = c_n + c_{-n}$, *and* $b_n = i(c_n - c_{-n})$.