(1) Let $G$ be a finite group and $n > 2$. Show that the number of elements in $G$ of order $n$ is even.

(2) Show that every group of even order has an element of order 2.

(3) If a group $G$ has a unique element $x$ of order 2 show that $x \in Z(G)$.
   (a) Show that any finitely-generated subgroup of $(\mathbb{Q}, +)$ (the additive group of the rationals) is cyclic. Use this to show that $(\mathbb{Q}, +)$ is not isomorphic to $(\mathbb{Q}, +) \oplus (\mathbb{Q}, +)$ (the direct sum of two copies of $\mathbb{Q}$.)
   (b) What happens if $\mathbb{Q}$ is replaced by $\mathbb{R}$ in both parts of (a)?

(4) Suppose $G$ is a group, $H \leq G$, and $K \leq G$. Give necessary and sufficient conditions for $H \cup K$ to be a group.

(5) Show that a group $G$ is the union of three proper subgroups if and only if there is an epimorphism from $G$ to Klein’s 4-group.

(6) (a) Suppose $H \leq G$. Show that $gHg^{-1}$ is a subgroup, and that $H \cong gHg^{-1}$.
   (b) Use (a) to show that in any group, $|xy| = |yx|$.

(7) (a) Prove that if $G/Z(G)$ is cyclic, then $G$ is abelian
   (b) Prove that if $Z(G)$ is maximal among abelian subgroups, then $G$ is abelian.

(8) Let $|G| = \infty$, and $[G : H] < \infty$. Show that $H$ intersects every infinite subgroup of $G$ nontrivially.

(9) Suppose $G$ is finite, $H \leq G$, and $G = \cup \{ xHx^{-1} \mid x \in G \}$. Show that $H = G$.

(10) Prove or give a counter example to each statement:
    (a) If every proper subgroup $H$ of a group $G$ is cyclic, then $G$ is cyclic.
    (b) If $H$ is a subgroup of an abelian group $G$, then both $H$ and the quotient group $G/H$ are abelian.
    (c) If $H$ is a normal abelian subgroup of a group $G$, and the quotient group $G/H$ is also abelian, then $G$ is abelian.
    (d) If $K \triangleleft H \triangleleft G$, then $K \triangleleft G$. 