

MthSc 208, Spring 2011 (Differential Equations)

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HW 2

Due Wednesday January 26th, 2010

- (1) Consider the initial value problem $y' = t + y$, $y(0) = 1$.
- (a) When computing a solution by hand using Euler's method, it is beneficial to arrange your work in a table, as shown below where the first step is computed. Continue with Euler's method using step-size $h = 0.1$ and complete all missing entries of the table.

k	t_k	y_k	$f(t_k, y_k) = t_k + y_k$	h	$f(t_k, y_k) \cdot h$
0	0.0	1.0	1.0	0.1	0.1
1	0.1	1.1			
2	0.2				
3	0.3				
4	0.4				
5	0.5				

- (b) The general solution of $y' = t + y$ is $y(t) = Ce^t - t - 1$. Using this, compute the actual value of $y(0.5)$.
- (2) Consider the initial value problem $y' = (1 + t)y$, $y(0) = -1$.
- (a) Use Euler's method to approximate $y(1)$, for step-size $h = 0.2$, and then for $h = 0.1$. Arrange your results in the tabular form as in the previous exercise.
- (b) Compute the actual value of $y(1)$ by solving the initial value problem $y' = (1 + t)y$, $y(0) = -1$ and plugging in $t = 1$.
- (3) Solve for t , and simplify whenever possible.
- (a) $3e^{-4t} = 5$
- (b) $2 = e^3 \cdot e^{2t}$
- (c) $t^2 = e^6$
- (d) $(\frac{4}{3})^{-t} = 7$
- (e) $e^{\frac{1}{3} \ln t} = 27$
- (f) $e^{-\frac{1}{3} \ln t} = 27$
- (4) Compute the following integrals:
- (a) $\int \frac{1}{2t} dt$
- (b) $\int \frac{1}{3-4t} dt$
- (5) Find the general solution of the following differential equations.
- (a) $y' = ty$
- (b) $ty' = -2y$
- (c) $y' = e^{t-y}$
- (6) Suppose that \$1200 is invested at a rate of 5%, compounded continuously.
- (a) Assuming no additional withdrawals or deposits, how much will be in the account after 10 years?
- (b) How long will it take the balance to reach \$5000?
- (7) Tritium is an isotope of hydrogen that is sometimes used as a biochemical tracer. Suppose that 100 mg of tritium decays to 80 mg in 4 hours. Determine its half-life.