MthSc 208, Spring 2011 (Differential Equations)
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HW 3
Due Monday January 31st, 2010

(1) Suppose a cold beer at 40°F is placed into a warm room at 70°F. Suppose 10 minutes later, the temperature of the beer if 48°F. Use Newton's law of cooling to find the temperature 25 minutes after the beer was placed into the room.

(2) A murder victim is discovered at midnight at the temperature of the body is recorded at 31°C. One hour later, the temperature of the body is 29°C. Assume that the surrounding air temperature remains constant at 21°C. Use Newton's law of cooling (the differential equation $T' = k(A-T)$) to calculate the victim's time of death (when his body temperature was 37°C).

(3) A parachutist of mass 60 kg free-falls from an airplane at an altitude of 5000 meters. He is subjected to an air resistance force proportional to his speed. Assume that the constant of proportionality is $r = 10$ kg/sec.
   (a) Find and solve the differential equation govering the altitude of the parachuter at time t seconds after the start of his free-fall.
   (b) Assuming he does not deploy his parachute, find his limiting velocity and how much time will elapse before he hits the ground (you may need to use a computer for this last part, a visual approximation from the appropriate graph is fine).

(4) In our model of air resistance, the resistance force has depended only on the velocity. However, for an object that drops a considerable distance, such as the parachutist in the previous exercise, there is a dependence on the altitude as well. It is reasonable to assume that the resistance force is proportional to air pressure, as well as to the velocity. Furthermore, to a first-order approximation, the air pressure varies exponentially with the altitude (i.e., it is proportional to e^{-ax}, where a is a constant and x is the altitude). Propose and justify (*but do not solve!*) a differential equation model for the velocity of a falling object subject to such a resistance force.

(5) For each of the first-order differential equations, decide whether it is linear or nonlinear. If the equation is linear, state whether it is homogeneous or inhomogeneous.
   (a) $y' = ky$
   (b) $y' = k(72 - y)$
   (c) $y' = y(4 - y)$
   (d) $y' = t + y$.
   (e) $3y' + 5y = 3\cos 2t$
   (f) $3y' + 5y = 3\cos 2y$
   (g) $y' = 4t^2y - \sin t$
   (h) $y' = 4ty^2 - \sin t$