MthSc 208, Spring 2011 (Differential Equations)
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HW 4
Due Friday February 4th, 2011

(1) Use the integrating factor method to find the general solution of the following differential equations.
  (a) $2y' - 3y = 5$
  (b) $y' + 2ty = 5t$
  (c) $ty' = 4y + t^4$

(2) Use the varation of parameters method to find the general solution of the following differential equation. Then find the particular solution satisfying the given initial condition.
  (a) $y' - 3y = 4, \quad y(0) = 2$
  (b) $y' + y = e^t, \quad y(0) = 1$
  (c) $y' + 2ty = 2t^3, \quad y(0) = -1$.

(3) A murder victim is discovered at midnight at the temperature of the body is recorded at 31°C, and it was discovered that the proportionality constant in Newton's law was $k = \ln(5/4)$. Assume that at midnight the surrounding air temperature $A(t)$ is 0°C, and is falling at a constant rate of 1°C per hour. At what time did the victim die? (Set $T(t) = 37$ and solve for t – use a computer or calculator for this part.) *Hint*: Letting $t = 0$ represent midnight will simplify your calculations.

(4) Consider Newton's law of cooling, but suppose that the ambient temperature varies sinusoidally with time, as in

$$T' = k(A \sin \omega t - T)\,.$$

  (a) Solve the homogeneous equation, $T_h' = -kT_h$.
  (b) The ODE above is not autonomous, so finding a particular solution T_p is a bit more difficult (there is no steady-state solution). However, it doesn't hurt to guess. As a first guess, substitute $T_p = C\cos\omega t + D \sin\omega t$ into the equation $T' + kT = kA\sin\omega t$ and equate coefficients of the sine and cosine terms, and show that

$$-\omega C + kD = kA \qquad \text{and} \qquad kC + \omega D = 0\,.$$

  (c) Solve the simultaneous equations in part (b), and determine the general solution to this ODE.
  (d) Give a qualitative physical description of what the particular solution T_p represents, and why. [Hint: Consider the long-term behavior of the temperature $T(t)$.]

(5) Suppose that the temperature T inside a mountain cabin behaves according to Newton's law of cooling, as in

$$\frac{dT}{dt} = \frac{1}{2}(A(t) - T)\,,$$

where t is measured in hours and the ambient temperature $A(t)$ outside the cabin varies sinusoidally with a period of 24 hours. At 6am, the ambient temperature outside is at a minimum of 40°, and at 6pm, the ambient temperature is at a maximum of 80°.
  (a) Adjust the differential equation above to model the sinusoidal nature of the ambient temperature.
  (b) Suppose that at noon the temperature inside the cabin is 50°. Solve the resulting initial value problem. [Hint: Use the formula you derived in part (c) of the previous problem! Letting $t = 0$ represent noon will simplify your calculations.]
  (c) Use a computer to sketch the graph of the temperature inside the cabin. On the same coordinate system, superimpose the plot of the ambient temperature outside the cabin. Comment on the appearance of the plot.

(6) Solve the differential equation $y' = 2y + 4$ four different ways:
  (a) Finding a constant solution, and writing $y(t) = y_h(t) + y_p(t)$.
  (b) Integrating factor
  (c) Variation of parameters

(d) Separation of variables