

MthSc 208, Spring 2011 (Differential Equations)

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HW 7

Due Friday February 18th, 2011

- (1) Solve the following initial value problems.
- (a) $y'' - y' - 2y = 0$, $y(0) = -1$, $y'(0) = 2$
 - (b) $y'' - 4y' - 5y = 0$, $y(1) = -1$, $y'(1) = -1$
 - (c) $y'' + 25y = 3$, $y(0) = 1$, $y'(0) = -1$
 - (d) $y'' - 2y' + 17y = 0$, $y(0) = -2$, $y'(0) = 3$
- (2) Find the general solution to the following 2nd order linear inhomogeneous ODEs, by solving the associated homogeneous equation, and then finding a constant (particular) solution.
- (a) $y'' + y' - 12y = 24$
 - (b) $y'' = -4y + 3$
- (3) As we've seen, to solve ODE of the form

$$y'' + py' + qy = 0, \quad p \text{ and } q \text{ constants}$$

we assume that the solution has the form e^{rt} , and then we plug this back into the ODE to get the *characteristic equation*: $r^2 + pr + q = 0$. Given that this equation has a double root $r = r_1$ (i.e., the roots are $r_1 = r_2$), show by direct substitution that $y = te^{r_1 t}$ is a solution of the ODE, and then write down the general solution. [Hint: If there's a double-root, then it must be $-\frac{p}{2}$. Why?]

- (4) Suppose that $z(t) = x(t) + iy(t)$ is a solution of

$$z'' + pz' + qz = Ae^{i\omega t}.$$

Substitute $z(t)$ into this equation above. Then compare (equate) the real and imaginary parts of each side to prove two facts:

$$x'' + px' + qx = A \cos \omega t$$

$$y'' + py' + qy = A \sin \omega t.$$

Write a sentence or two summarizing the significance of this result.

- (5) Solve the following initial value problems using the method of undetermined coefficients.
- (a) $y'' + 3y' + 2y = -3e^{-4t}$, $y(0) = 1$, $y'(0) = 0$
 - (b) $y'' + 2y' + 2y = 2 \cos 2t$, $y(0) = -2$, $y'(0) = 0$
 - (c) $y'' + 4y' + 4y = 4 - t$, $y(0) = -1$, $y'(0) = 0$
 - (d) $y'' - 2y' + y = t^3$, $y(0) = 1$, $y'(0) = 0$
- (6) Solve the following first order differential equations using the method of undetermined coefficients.
- (a) $y' - 3y = 5e^{2t}$
 - (b) $y' + 2y = 3t$
 - (c) $T' = k(-t - T)$
 - (d) $T' = k(A \sin \omega t - T)$