(1) If \( y_f(t) \) is a solution of
\[ y'' + py' + qy = f(t) \]
and \( y_g(t) \) is a solution of
\[ y'' + py' + qy = g(t), \]
show that \( z(t) = \alpha y_f(t) + \beta y_g(t) \) is a solution of
\[ y'' + py' + qy = \alpha f(t) + \beta g(t), \]
where \( \alpha \) and \( \beta \) are any real numbers, by plugging it into the ODE.

(2) Find the general solution to the following 2nd order linear inhomogeneous ODEs.

(a) \( y'' + 2y' + 2y = 2 + \cos 2t \)
(b) \( y'' + 25y = 2 + 3t + 4 \cos 2t \)
(c) \( y'' - y = t - e^{-t} \)

(3) (a) Find the general solution of \( y'' + 3y' + 2y = te^{-4t} \). (Look for a particular solution of the form \( y_p = (at + b)e^{-4t} \).)

(b) Use a similar approach as above to find a solution to the differential equation \( y'' + 2y' + y = t^2e^{-2t} \).

(4) Find the general solution of \( y'' + 2y' + 2y = e^{-2t}\sin t \). (Look for a particular solution of the form \( y_p = e^{-2t}(a \cos t + b \sin t) \).)

(5) For the following exercises, rewrite the given function in the form
\[ y = A \cos(\omega t - \phi) = A \cos \left( \omega \left( t - \frac{\phi}{\omega} \right) \right), \]
and then plot the graph of this function.

(a) \( y = \cos 2t + \sin 2t \)
(b) \( y = \cos t - \sin t \)
(c) \( y = \cos 4t + \sqrt{3} \sin 4t \)
(d) \( y = -\sqrt{3} \cos 2t + \sin 2t \).

(6) Consider the undamped oscillator
\[ mx'' + kx = 0, \quad x(0) = x_0, \quad x'(0) = v_0. \]

(a) Write the particular solution of this initial value problem in the form \( x(t) = a \cos \omega t + b \sin \omega t \) (i.e., determine \( a, b, \) and \( \omega \)).

(b) Write your solution in the form \( x(t) = A \cos(\omega t - \phi) \) (i.e., determine \( A \)).