MthSc 208, Spring 2011 (Differential Equations)
Dr. Matthew Macauley
HW 9
Due Monday February 28th, 2011

(1) A 0.1-kg mass is attached to a spring having a spring constant 3.6 kg/s^2. The system is allowed to come to rest. Then the mass is given a sharp tap, imparting an instantaneous downward velocity of 0.4 m/s. If there is no damping present, find the amplitude A, frequency ω, and phase-shift ϕ, of the resulting motion.
 (a) Let $x = 0$ be the position of the spring *before* the mass was hung from it. Find $x(0)$.
 (b) Solve this initial value problem and plot the solution.

(2) A spring-mass system is modeled by the equation

$$x'' + \mu x' + 4x = 0 .$$

 (a) Show that the system is critically damped when $\mu = 4$ kg/s.
 (b) Suppose that the mass is displaced upward 2 m and given an initial velocity of 1 m/s. Use a computer (i.e., WolframAlpha) to comute the solution for $\mu = 4$, 4.2, 4.4, 4.6, 4.8, 5. Plot all of the solution curves on one figure. What is special about the critically damped solution in comparison to the other solutions?
 (c) On a new set of axes, repeat part (b) using $\mu = 4$, 3.9, and 3.
 (d) Explain why would you want to adjust the spring on a screen door so that it was critically damped.

(3) The function $x(t) = \cos 6t - \cos 7t$ has mean frequency $\bar{\omega} = 13/2$ and half difference $\delta = 1/2$. Thus,

$$\cos 6t - \cos 7t = \cos\left(\frac{13}{2} - \frac{1}{2}\right)t - \cos\left(\frac{13}{2} + \frac{1}{2}\right)t = 2\sin\frac{1}{2}t\,\sin\frac{13}{2}t .$$

Plot the graph of $x(t)$, and superimpose the "envelope" of the beats, which is the slow frequency oscillation $y(t) = \pm 2\sin(1/2)t$. Use different line styles or colors to differentiate the curves.

(4) Plot the given function on an appropriate time interval. Use the technique of the previous exercise to superimpose the plot of the envelope of the beats in a different line style and/or color.
 (a) $\cos 9t - \cos 10t$
 (b) $\sin 11t - \sin 10t$

(5) Let $\omega_0 = 11$. Use a computer to plot the graph of the solution

$$x(t) = \frac{\cos\omega t - \cos\omega_0 t}{\omega_0^2 - \omega^2}$$

for $\omega = 9$, 10, 10.5, 10.9, and 10.99 on the time interval $[0, 24]$. (Okay to just print this out and attach it). Explain how these solutions approach the resonance solution as $\omega \to \omega_0$. *Hint*: Put the equation above in the form $x(t) = A\sin\delta t\,\sin\bar{\omega}t$, and use this result to justify your conclusion.