MthSc 208, Spring 2011 (Differential Equations)
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HW 10
Due Thursday March 3rd, 2011

(1) Solve the following differential equations.
  (a) $y' = -3y$
  (b) $2y' = t + 6y$
  (c) $2y' = t^2 + 6y$
  (d) $y'' + 4y = 0$
  (e) $y'' = -9y + 12$.

(2) For each system below, write it as $\mathbf{Ax} = \mathbf{b}$. Find all solutions, and sketch the graph of the lines in each system on the same axis. Are the resulting lines intersecting, parallel, or coincident?
  (a) $x_1 + 3x_2 = 0, \quad 2x_1 - x_2 = 0$
  (b) $-x_1 + 2x_2 = 4, \quad 2x_1 - 4x_2 = -6$
  (c) $2x_1 - 3x_2 = 4, \quad x_1 + 2x_2 = -5$
  (d) $3x_1 - 2x_2 = 0, \quad -6x_1 + 4x_2 = 0$
  (e) $2x_1 - 3x_2 = 6, \quad -4x_1 + 6x_2 = -12$

(3) For each part, find the determinant, eigenvalues and eigenvectors of the given matrix. If the matrix is invertible, find its inverse.
  (a) $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$ (b) $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$ (c) $\mathbf{A} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$
  (d) $\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}$ (e) $\mathbf{A} = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix}$ (f) $\mathbf{A} = \begin{pmatrix} 5/4 & 3/4 \\ -3/4 & -1/4 \end{pmatrix}$

(4) For each problem below, find the eigenvalues of the given matrix, and then describe how the nature of the eigenvalue (e.g., positive/negative, complex, repeated, etc.) depends on the parameter α.
  (a) $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & \alpha \end{pmatrix}$ (b) $\mathbf{A} = \begin{pmatrix} 1 & -\alpha \\ 2\alpha & 3 \end{pmatrix}$

(5) In this problem we will show that $\lambda = 0$ is an eigenvalue of a matrix $\mathbf{A}$ if and only if $\det(\mathbf{A}) = 0$.
  (a) Show that if $\lambda = 0$ is an eigenvalue of $\mathbf{A}$, then $\det(\mathbf{A}) = 0$.
  (b) Show that if $\det(\mathbf{A}) = 0$, then $\lambda = 0$ is an eigenvalue of $\mathbf{A}$.