MthSc 208, Spring 2011 (Differential Equations)
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HW 14
Due Thursday, March 31st, 2011

(1) Find the inverse Laplace transform of the following functions.
  (a) $Y(s) = \dfrac{2}{3-5s}$
  (b) $Y(s) = \dfrac{1}{s^2+4}$
  (c) $Y(s) = \dfrac{5s}{s^2+9}$
  (d) $Y(s) = \dfrac{3}{s^2}$
  (e) $Y(s) = \dfrac{3s+2}{s^2+25}$
  (f) $Y(s) = \dfrac{2-5s}{s^2+9}$
  (g) $Y(s) = \dfrac{s}{(s+2)^2+4}$
  (h) $Y(s) = \dfrac{3s+2}{s^2+4s+29}$
  (i) $Y(s) = \dfrac{2s-2}{(s-4)(s+2)}$
  (j) $Y(s) = \dfrac{3s^2+s+1}{(s-2)(s^2+1)}$

(2) Use the Laplace transform to solve the following initial value problems.
  (a) $y' - 4y = e^{-2t}t^2, \;\; y(0) = 1$
  (b) $y'' - 9y = -2e^t, \;\; y(0) = 0, \;\; y'(0) = 1$

(3) Find the Laplace transform of the given functions.
  (a) $3H(t-2)$
  (b) $(t-2)H(t-2)$
  (c) $e^{2(t-1)}H(t-1)$
  (d) $H(t-\pi/4)\sin 3(t-\pi/4)$
  (e) $t^2H(t-1)$
  (f) $e^{-t}H(t-2)$

(4) In this exercise, you will examine the effect of shifts in the time domain on the Laplace transform (graphically).
  (a) Sketch the graph of $f(t) = \sin t$ in the time domain. Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}(s)$. Sketch the graph of F in the s-domain on the interval $[0,2]$.
  (b) Sketch the graph of $g(t) = H(t-1)\sin(t-1)$ in the time domain. Find the Laplace transform $G(s) = \mathcal{L}\{g(t)\}(s)$. Sketch the graph of G in the s-domain on the interval $[0,2]$ on the same axes used to sketch the graph of F.
  (c) Repeat the directions in part (b) for $g(t) = H(t-2)\sin(t-2)$. Explain why engineers like to say that "a shift in the time domain leads to an attenuation (scaling) in the s-domain."

(5) Use the Heaviside function to concisely write each piecewise function.
  (a) $f(t) = \begin{cases} 5 & 2 \le t < 4; \\ 0 & \text{otherwise} \end{cases}$
  (b) $f(t) = \begin{cases} 0 & t < 0; \\ t & 0 \le t < 3 \\ 4 & t \ge 3 \end{cases}$
  (c) $f(t) = \begin{cases} 0 & t < 0; \\ t^2 & 0 \le t < 2 \\ 4 & t \ge 2 \end{cases}$

(6) Find the inverse Laplace transform of each function. Create a piecewise definition for your solution that doesn't use the Heavyside function.

(a) $F(s) = \dfrac{e^{-2s}}{s+3}$

(b) $F(s) = \dfrac{1 - e^{-s}}{s^2}$

(c) $F(s) = \dfrac{e^{-s}}{s^2+4}$