

MthSc 208, Spring 2011 (Differential Equations)

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HW 16

Due Friday April 8th, 2011

(1) Solve the following differential equations:

(a) $y' - 2y = 0$

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(b) $y' - 2y = e^{3t}$

(c) $y'' - 4y = 0$

(d) $y'' + 4y = 0$

(e) $y'' + 4y' + 3y = 10$.

(2) The function

$$f(x) = \begin{cases} 0 & -\pi \leq x < -\pi/2, \\ 1 & -\pi/2 \leq x < \pi/2, \\ 0 & \pi/2 \leq x \leq \pi, \end{cases}$$

can be extended to be periodic of period 2π . Sketch the graph of the resulting function, and compute its Fourier series.

(3) The function

$$f(t) = |x|, \quad \text{for } x \in [-\pi, \pi]$$

can be extended to be periodic of period 2π . Sketch the graph of the resulting function, and compute its Fourier series.

(4) The function

$$f(x) = \begin{cases} 0 & -\pi \leq x < 0, \\ x & 0 \leq x \leq \pi, \end{cases}$$

can be extended to be periodic of period 2π . Sketch the graph of the resulting function, and compute its Fourier series.

(5) Consider the 2π -periodic function defined by

$$f(x) = \begin{cases} x^2 & -\pi \leq x < \pi, \\ f(x - 2k\pi), & -\pi + 2k\pi \leq x < \pi + 2k\pi. \end{cases}$$

Sketch this function (at least for $k = -2, -1, 0, 1, 2$) and compute its Fourier series.

(6) Find the Fourier series of the following functions *without* computing any integrals.

(a) $f(x) = 2 - 3 \sin 4x + 5 \cos 6x$,

(b) $f(x) = \sin^2 x$ [*Hint*: Use a standard trig identity.]

(7) Determine which of the following functions are even, which are odd, and which are neither even nor odd:

(a) $f(t) = x^3 + 3x$.

(b) $f(t) = x^2 + |x|$.

(c) $f(t) = e^x$.

(d) $f(t) = \frac{1}{2}(e^x + e^{-x})$.

(e) $f(t) = \frac{1}{2}(e^x - e^{-x})$.