

Week 5 summary:

- 2nd order linear ODE: $y'' + p(t)y' + q(y)y = f(t)$

Homogeneous if $f(t) = 0$.

General solution: $y(t) = y_h(t) + y_p(t) = C_1 y_1(t) + C_2 y_2(t) + y_p(t)$

- Constant coefficients: $y'' + py' + qy = 0$

Assume $y(t) = e^{rt}$, plug back in and solve for r .

$$\text{Get } e^{rt}(r^2 + pr + q) = 0$$

3 cases: (i) $r_1 \neq r_2$ real

$$y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

$$(ii) \quad r_1 = r_2$$

$$y(t) = C_1 e^{r_1 t} + C_2 t e^{r_1 t}$$

$$(iii) \quad r_{1,2} = a \pm bi$$

$$y(t) = e^{at}(A \cos bt + B \sin bt).$$

- Inhomogeneous equations: $y'' + py' + qy = f(t)$, $f(t) \neq 0$.

Guess that $y_p(t)$ "has the same form" as $f(t)$.

This is the "method of undetermined coefficients"

$f(t)$	$y_p(t)$
e^{rt}	$a e^{rt}$
$\star \cos wt \text{ or } \sin wt$	$\star a \cos wt + b \sin wt$
$\star \text{degree-}n \text{ polynomial}$	$\star \text{degree-}n \text{ polynomial}$
$\star e^{rt} \cos wt \text{ or } e^{rt} \sin wt$	$\star e^{rt} (a \cos wt + b \sin wt)$
$\star \text{linear combination of above functions}$	$\star \text{linear combination of above functions}$