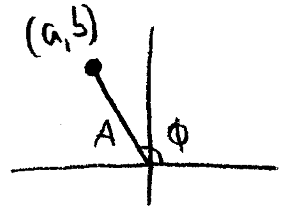
MATHSC 208, Spring 2011

Week 6 summary:

- Simple harmonic motion: $x'' = -w^2 x$ has solution

$$x(t) = a\cos wt + b\sin wt = A\cos\left(w\left(t-\frac{\phi}{w}\right)\right), \qquad A = \sqrt{a^2+b^2}, \quad \phi = \text{"}\tan^{-1}(b/a)\text{"}$$



- General harmonic motion: $m x'' + 2c x' + w_0^2 x = f(t)$

$$\underbrace{m x''}_{\text{Newton's } 2^{nd}} + \underbrace{2c x'}_{\text{damping force}} + \underbrace{w_0^2 x}_{\text{spring force}} = \underbrace{f(t)}_{\text{driving force.}}$$

* with damping ($c \neq 0$, say $f(t) = 0$): Roots $r_{1,2} = -c \pm \sqrt{c^2 - w_0^2}$

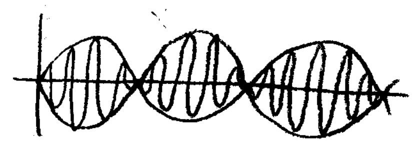
Case 1: $c > w_0$ overdamped $\quad x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

Case 2: $c < w_0$ underdamped $\quad x(t) = e^{-ct}(a\cos w_0 t + b\sin w_0 t)$

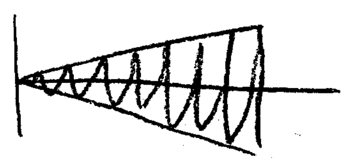
Case 3: $c = w_0$ critically damped $\quad x(t) = C_1 e^{rt} + C_2 t e^{rt}$

* forced harmonic motion: $(f(t) \neq 0)$ e.g., $x'' + w_0^2 x = A\cos wt \quad x(0) = x'(0) = 0$.

Case 1: $w \neq w_0$: $x(t) = \dfrac{A}{2\delta\bar{w}} \sin\delta t \sin\bar{w} t$



Case 2: $w = w_0$: $x(t) = \dfrac{A}{2w_0} t \sin w_0 t$



- Basic linear algebra:

* A system of linear equations $\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$

can be written as $\boxed{A\vec{x} = \vec{b}}$, or $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

* A has an inverse iff $\det A := a_{11}a_{22} - a_{12}a_{21} \neq 0$; $\quad AA^{-1} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

The inverse of A is $\dfrac{1}{\det A}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$, and $\vec{x} = A^{-1}\vec{b}$ in this case