Week 8 summary:

- The system \( \dot{x} = Ax \) has general solution \( \dot{x}(t) = C_1 e^{\lambda_1 t} \hat{v}_1 + C_2 e^{\lambda_2 t} \hat{v}_2 \), where \( \lambda_{1,2} \) are the eigenvalues of \( A \), and \( \hat{v}_{1,2} \) are the eigenvectors.

- To solve \( \dot{x'} = Ax + b \), find the steady-state solution \( \bar{x}_{st}(t) \) (set \( \dot{x'} = 0 \)), then solve the homogeneous system (set \( b = 0 \)). The general solution is \( \dot{x}(t) = \bar{x}_h(t) + \bar{x}_{st}(t) \).

- Phase portraits: Plotting \( x_2 \) vs. \( x_1 \). The "eigenvector lines" contain straight line solutions.

Examples:

\[
\begin{align*}
\lambda_1 > \lambda_2 > 0 & \\
\lambda_1 < \lambda_2 < 0 & \\
\lambda_1 < 0 < \lambda_2 &
\end{align*}
\]