MTHSc 208: Spring 2011

Week 12 summary:

- For $Per_{2L} = \{2L\text{-periodic functions}\}$, define

  $\langle f(x), g(x) \rangle = \frac{1}{L}\int_{-L}^{L} f(x)\,g(x)\,dx$. This allows us to compute

  the <u>Fourier series</u>: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$

  $$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx, \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

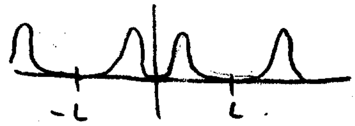
- <u>Even functions</u>: $f(x) = f(-x)$.
  - Symmetric about y-axis
  - Fourier series contains only cosines
  - $\int_{-L}^{L} f(x)\,dx = 2\int_0^L f(x)\,dx$

  <u>Odd functions</u>: $f(x) = -f(-x)$.
  - Symmetric about origin
  - Fourier series contains only sines
  - $\int_{-L}^{L} f(x)\,dx = 0$

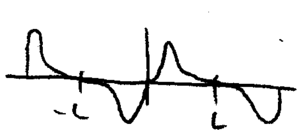
- <u>Fourier cosine and sine series</u>:

  Start with a function $f(x)$ on $[0, L]$, e.g., 

  - <u>Fourier cosine series</u> is the Fourier series of the <u>even</u> extension,

    e.g.,  $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$, $a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$.

  - <u>Fourier sine series</u> is the Fourier series of the <u>odd</u> extension,

    e.g.,  $f(x) = b_n \sin\left(\frac{n\pi x}{L}\right)$, $b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$.

- <u>Complex Fourier series</u>: $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{-inx} = c_0 + \sum_{n=1}^{\infty} (c_n e^{-inx} + c_{-n} e^{inx})$

$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx}$ (for 2π-periodic functions)

$a_n = c_n + c_{-n}$

$b_n = i(c_n - c_{-n})$

$c_n = \frac{a_n - ib_n}{2}$ $\quad c_{-n} = \frac{a_n + ib_n}{2}$.