MTHSc 208: Spring 2011

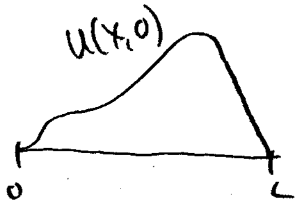
Week 13 summary:

- Parseval's identity: $\langle f(x), f(x)\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi}(f(x))^2\,dx = \frac{1}{2}a_0^2 + \sum_{n=1}^{\infty} a_n^2 + b_n^2$

  Can be used for computing sums! e.g., $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

- Partial differential equations (PDE's): Equations involving a multivariate function and its partial derivatives.

- Heat equation: $u_t = c^2 u_{xx}$, $u(x,t)$ = temp. at pos. x, time t.



  * Boundary conditions: e.g., $u(0,t) = u(L,t) = 0$
  * Initial condition: e.g., $u(x,0) = h(x)$ (initial temp of bar)

- Solving the heat equation: $u_t = c^2 u_{xx}$
  * Assume $u(x,t) = f(x)\,g(t)$. Compute u_t, u_{xx}, "zero-boundary conditions"
  * Plug back in and separate variables; set equal to λ.
  * Solve ODE's for $g(t)$ and $f(x)$.
  * Get a sol'n $u_n(x,t) = f_n(x)\,g_n(t)$ for each n.
  * Gen'l sol'n is $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$ (superposition)
  * Plug in $t=0$ & use initial condition (may require finding a Fourier sine or cosine series.