Week 14 summary:

- **Boundary conditions** for the heat equation:
  
  * **Dirichlet** specify the value, e.g., \( U(0,t) = T_1, \ U(L,t) = T_2. \)
  
  * **von Neumann** specify the derivative, e.g., \( U_x(0,t) = 0, \ U_x(L,t) = 0. \)

  This represents insulated endpoints.

- **Wave equation**: \( U_{tt} = c^2 U_{xx} \)

  Boundary conditions:
  
  \( U(0,t) = U(L,t) = 0 \)

  Initial conditions:
  
  \( U(x,0) = h_1(x) \) "initial position"

  \( U_t(x,0) = h_2(x) \) "initial velocity"

  Main difference: \( g(t) = \alpha \cos(cnt) + \beta \sin(cnt) \) instead of \( A e^{-c^2n^2t}. \)

- **PDEs in 2 dimensions**

  * **Heat equation**: \( U_t = c^2(U_{xx} + U_{yy}) \)
  
  * **Wave equation**: \( U_{tt} = c^2(U_{xx} + U_{yy}) \)

- **PDEs in n dimensions**

  * **Heat equation**: \( U_t = \nabla^2 U \)
  
  * **Wave equation**: \( U_{tt} = \nabla^2 U \)

  Where \( \nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \cdots + \frac{\partial^2 U}{\partial x_n^2} \) the "Laplacian" of \( U. \)
Harmonic functions: \( \nabla^2 u = 0 \)

Harmonic functions

\( \nabla^2 u = 0 \)

\[ \text{steady-state solns to the heat eqn} \]

"Flat" functions; soap bubble or plastic wrap surfaces