MTHSC 208: Spring 2011

Week 14 summary:

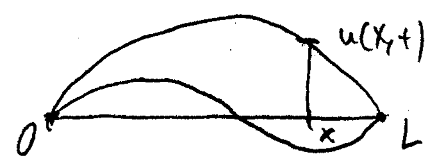
- Boundary conditions for the heat equation:

  * Dirichlet: specify the value, e.g., $u(0,t) = T_1$, $u(L,t) = T_2$.

  * von Neumann: specify the derivative, e.g., $u_x(0,t) = 0$, $u_x(L,t) = 0$.

    This represents insulated endpoints.

- Wave equation: $u_{tt} = c^2 u_{xx}$



  Boundary conditions: $u(0,t) = u(L,t) = 0$

  Initial conditions: $u(x,0) = h_1(x)$ "initial position"

  $u_t(x,0) = h_2(x)$ "initial velocity"

  Main difference: $g(t) = a\cos(cnt) + b\sin(cnt)$ instead of $Ae^{-c^2n^2t}$.

- PDEs in 2 dimensions

  * Heat equation: $u_t = c^2(u_{xx} + u_{yy})$

  * Wave equation: $u_{tt} = c^2(u_{xx} + u_{yy})$

- PDEs in n dimensions

  * Heat equation: $u_t = \nabla^2 u$

  * Wave equation: $u_{tt} = \nabla^2 u$

  where $\nabla^2 u = \frac{\partial^2 u}{\partial x_1^2} + \cdots + \frac{\partial^2 u}{\partial x_n^2}$, the "Laplacian" of u.

- Harmonic functions: $\nabla^2 u = 0$

$\boxed{\text{Harmonic functions } \nabla^2 u = 0} \xleftrightarrow{\text{same}} \boxed{\text{steady-state sol'ns to the heat eq'n}} \xleftrightarrow{\text{same}} \boxed{\text{"Flat" functions; soap bubble or plastic wrap surfaces}}$