Week 15 summary:

• Solving Laplace equation in 2D: \( U_{xx} + U_{yy} = 0 \)

Separate variables, do it piece-by-piece, use superposition

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\]

• 2D heat equation: \( U_t = \nabla^2 U \)

Solution is \( u(x, y, t) = U_h(x, y, t) + U_\delta(x, y) \)

where \( U_h(x, y, t) \) solves the homogeneous eq'n (zero-boundary condition)

• Solving the 2D heat and wave equations (homogeneous)

  * Assume \( U(x, y, t) = f(x, y) g(t) \), i.e. separate variables.
  * Get the Helmholtz eq'n for \( f: \) \( \nabla^2 f + \lambda f, \ 9 = -(n^2 + m^2) \).
  * The general solution is \( \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \).
  * Use the initial condition(s): plug in \( t = 0 \) and equate coefficients.