

**MthSc 208: Differential Equations (Spring 2011)**  
**In-class Worksheet 4d: Systems of differential equations (repeated eigenvalues)**

NAME:

Consider the system of differential equations:  $\begin{cases} x_1' = -x_1 - x_2 \\ x_2' = x_1 - 3x_2 \end{cases}$

1. Write this in matrix form,  $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$ .
2. Knowing that  $\mathbf{A}$  has a repeated eigenvalue,  $\lambda_{1,2} = -2$ , and one eigenvector,  $\mathbf{v}_1 = (1, 1)$ , write down a solution  $\mathbf{x}_1(t)$  to  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ .
3. To find a second solution, assume that  $\mathbf{x}_2(t) = te^{\lambda t}\mathbf{v} + e^{\lambda t}\mathbf{w}$ . Plug this back into  $\mathbf{x}' = \mathbf{A}\mathbf{x}$  and equate coefficients (of  $te^{-\lambda t}$  and  $e^{\lambda t}$ ) to get a system of two equations, involving  $\mathbf{v}$ ,  $\mathbf{w}$ , and  $\mathbf{A}$ .

